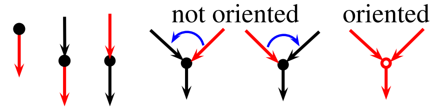
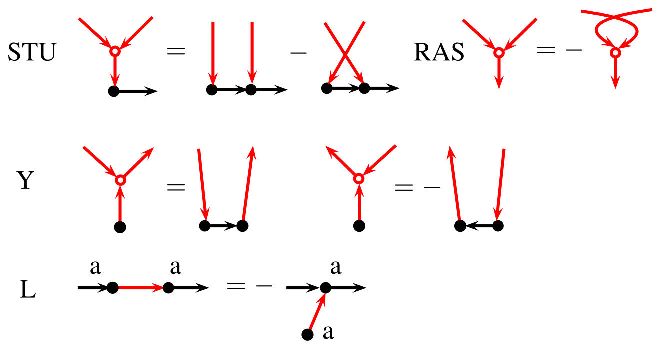
Red-black diagrams and cyclic words

Vertices



$\mathcal{T}^{RB} = \{$RB-trees, labeled snakes$\}/RAS, STU, Y, L, RIHX$

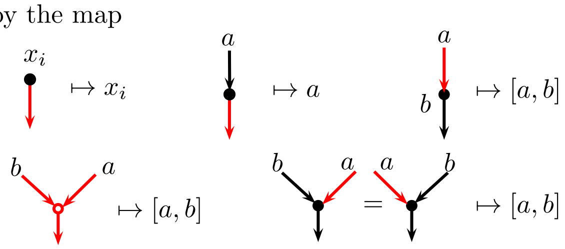
$\mathcal{D}^{RB} = \{$RB-diagrams, labeled snakes$\}/RAS, STU, Y, L, RIHX$



Remark 1. If snakes are labeled by letters $x = \{x_i\}$ then

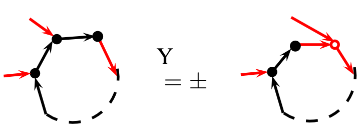
$$T^{RB} \cong FL(x)$$

by the map



STU is Jacobi identity, all other relations are bracket antisymmetry.

Remark 2. Any RB-diagram where wheel is not completely black is equivalent to a lin. comb. of completely red diagrams, with labeled leaves.



Use Y to shorten snakes starting from heads. If a snake has no head, it is the complete wheel.

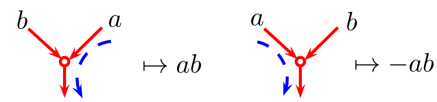
Def. With $FL(x)$ the free Lie algebra on letters x, $FA(x)$ the free associative algebra on x,

$$CW(x) = FA(FL(x))/w_1 w_2 \cdots w_k = w_2 \cdots w_k w_1$$

$$DW(x) = CW(x)/w_1 w_2 \cdots w_k = (-1)^{1+\sum deg w_i} w_k \cdots w_2 w_1$$

Conj. $\mathcal{D}^{RB}$/wheel reversal $\cong DW(x)$

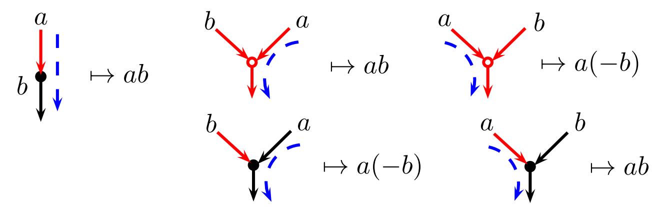
Pf 1. By remark 2 can assume diagram completely red (no, almost!). Need to extend the map in remark 1 to vertices on wheel. If with an orientation get $\varepsilon w_1 \dots w_k$, after reversing get



$(-1)^k \varepsilon w_k \dots w_1$.

Pf 2. Trees are words in $FL(x)$ by Remark 1.

Need to extend the map in remark 1 to vertices on wheel.



But it does not satisfy STU!