Course Idea: 12 Definitions of the Alexander Polynomial.

1. The "cars" matrix: knots, R-moves, rotation numbers, proof of invariance.
2. pi_1 and fox derivatives.
3. H_1.
4. Seifert.
5. Burau.
6. The Moscow definition.
7. Conway.
8. Meta-monoids.
9. w.
10. Kontsevich.
11. Hopf/Kerler.
12. Surprise!

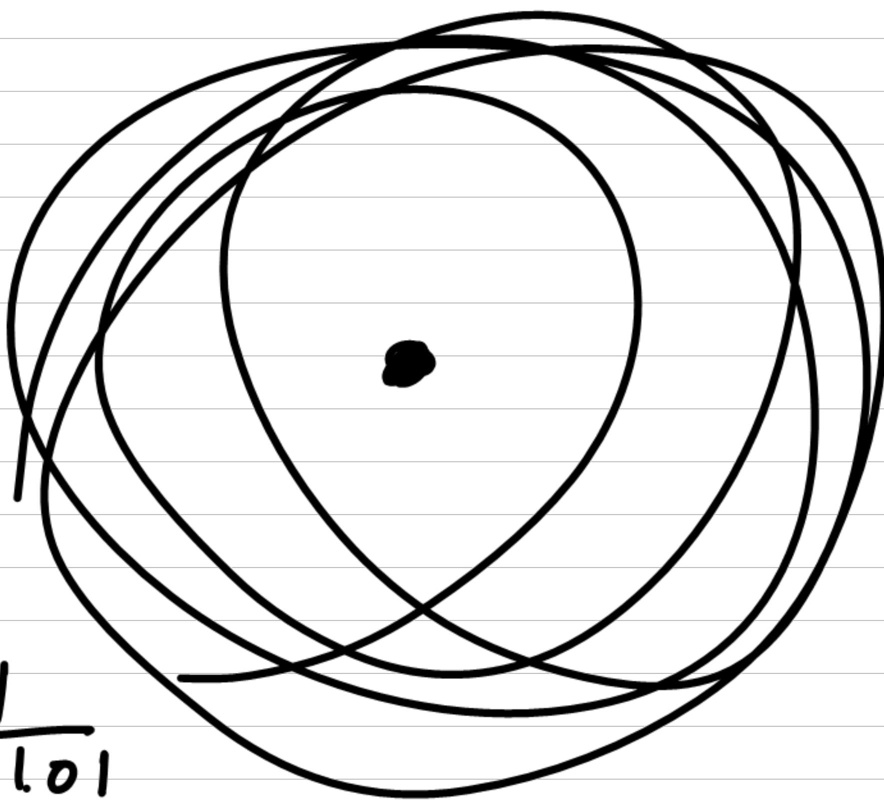
In math, we often care for things for how well-connected they are. In themselves, 57 and 1729 and 196884 are just members of an infinitely long dull and monotone procession of "numbers". Yet the Theory of Numbers talks to nearly everything in mathematics, and everything talks to it. Knots are likewise dull, and Knot Theory is likewise interesting, and within Knot Theory the Alexander Polynomial plays a special role: it is arguably the most successful "Knot Invariant", it talks to everything, and everything talks to it.

In this class we will cover 12 of the 50 or so definitions of the Alexander Polynomial and discuss how they are related to formal probability and determinants, fundamental groups, and Fox derivatives, homology, covering spaces and finitely presented modules, Seifert surfaces and linking forms, braids and their Burau representation, exterior algebras and the Berezin integral, skein relations, tangles and meta-monoids, finite type invariants and the Kontsevich integral, 2-knots in 4D and the w-expansion and Hopf algebras and algebraic knot theory.

We will aim to implement in Mathematica almost everything that we will talk about, so this class is also about turning sophisticated mathematics into concise and effective code.

Prerequisites: Excellent grasp of everything in Core Algebra I (MAT1100) and in Core Topology (MAT1301), and no fear of computers.

Evaluation: Near-weekly problem sets, a possible full day of student lectures at the end.



$\frac{1}{r}$

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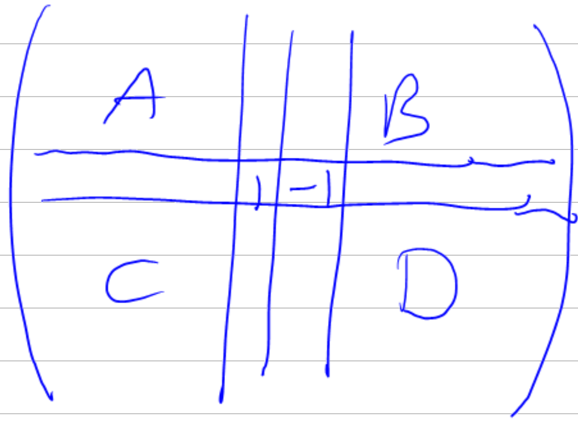


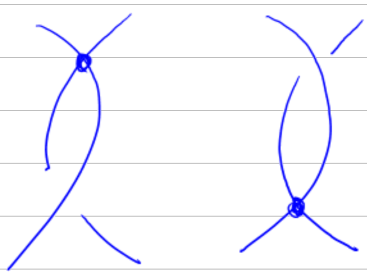

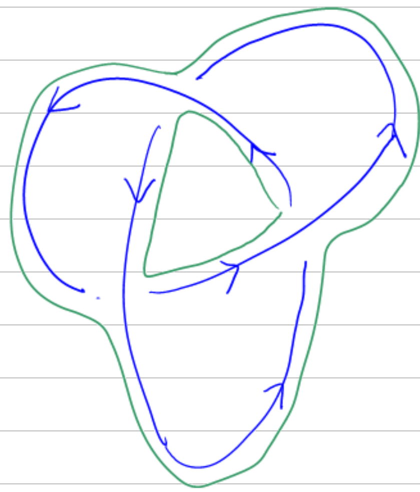

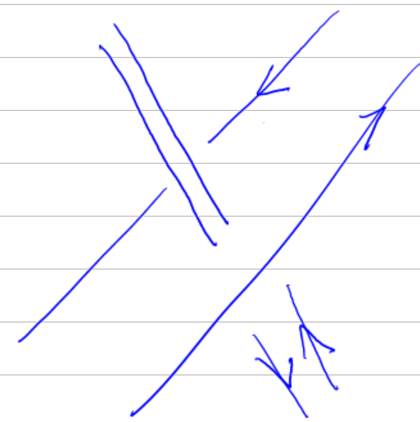

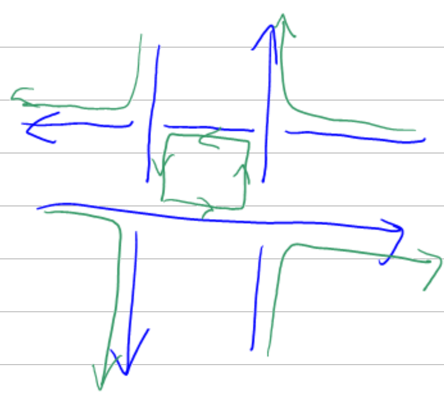

Find a seifert-slides invariant face formula for Θ

Title. A Seifert Dream

Abstract. Given a knot K with a Seifert surface \Sigma, I dream that the well-known Seifert linking form Q, a quadratic on H1xH1, has a docile local perturbation P_\eps such that the formal Gaussian integral of \exp(Q+P_\eps) is an invariant of K.

In my talk I will explain what the above means, why this dream is oh so sweet, and why it is in fact closer to a plan than to a delusion.