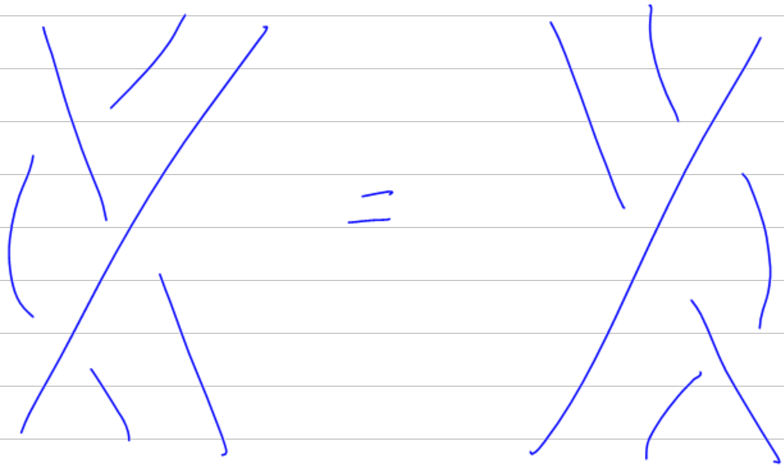
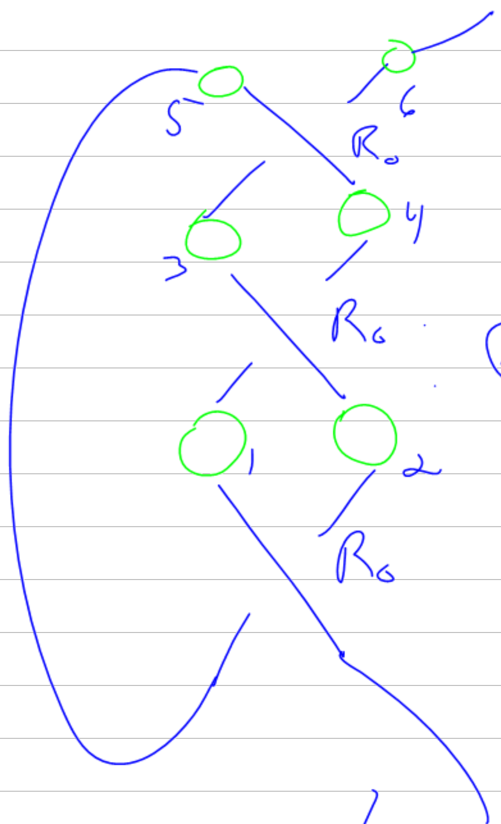
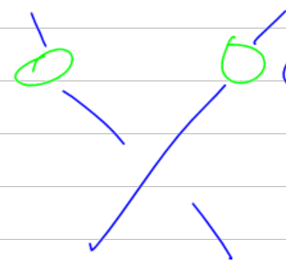
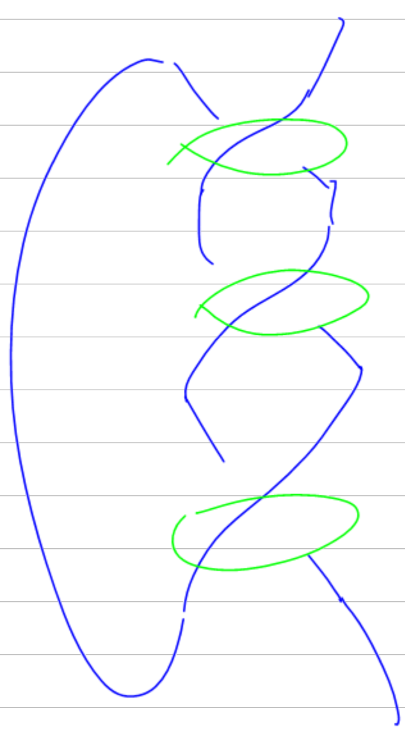


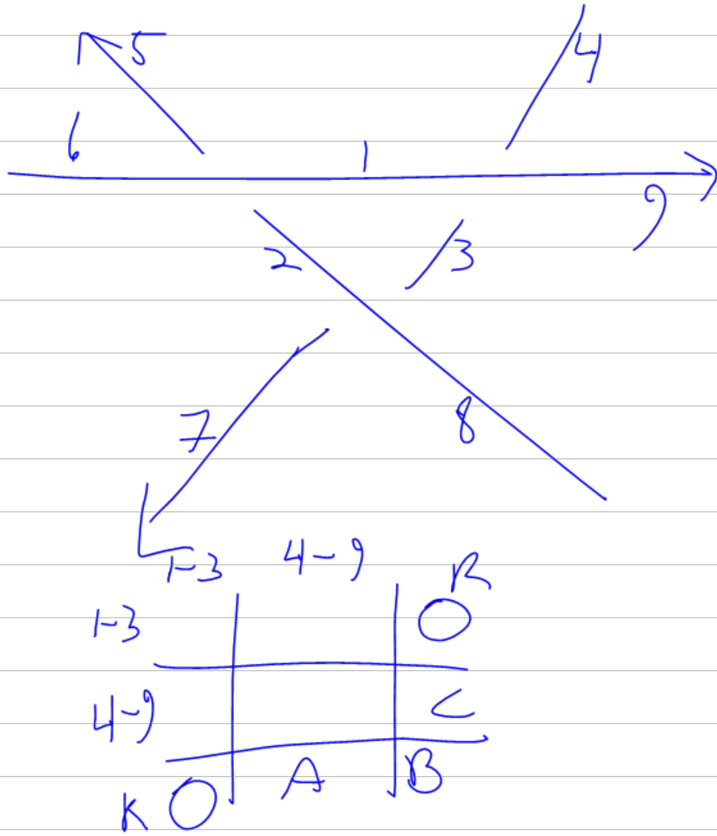


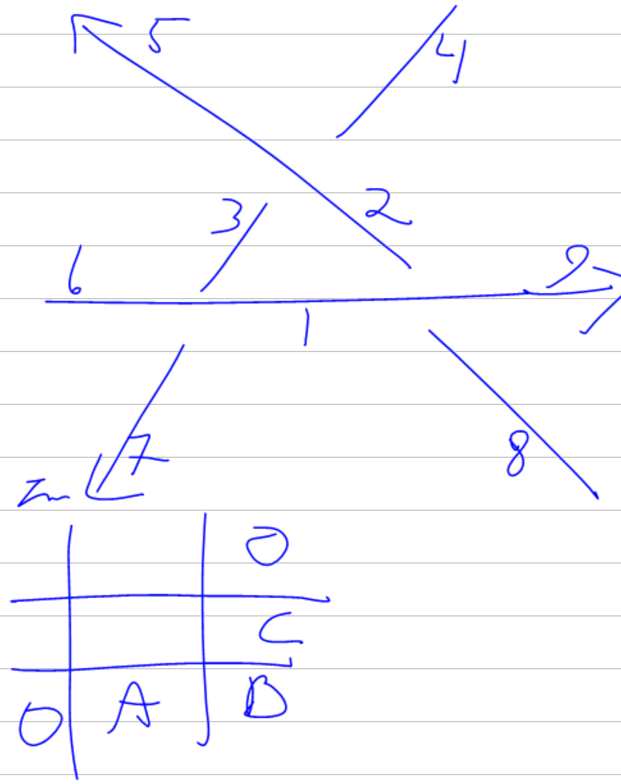

$$\mathbb{Q}[p_i, x_i : i=1,\dots,6] \longrightarrow \mathbb{Q}[p,x] \xrightarrow{ev_{0,0}} \mathbb{Q}$$



$e^{\pi_i p_j + \xi_i x_j}$

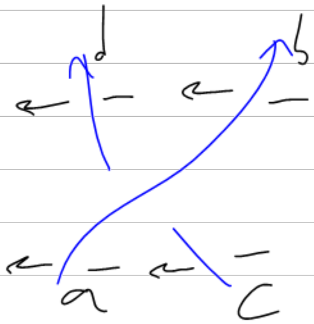


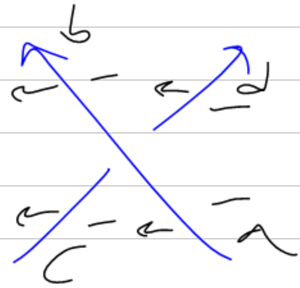



$$(EAF)^{-1} = F^{-1}A^{-1}E^{-1}$$

$$0 = \partial_x x^{-1}x = \partial_x(x^{-1}) + T^{-1}$$



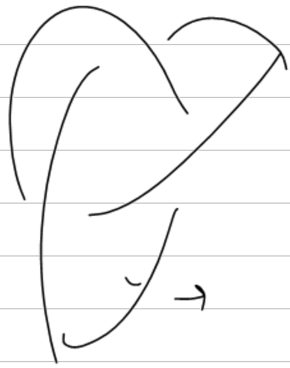

| | | a | b | c | d |
|---|---|---|---|---|---|
| rel 1 | $ab^{-1}$ | 1 | $-1$ | 0 | 0 |
| rel 2 | $cbd^{-1}b^{-1}$ | 0 | $T-1$ | 1 | $-T$ |
| rel 1 | $ab^{-1}$ | 1 | $-1$ | | |
| rel 2 | $cb^{-1}d^{-1}b$ | 0 | $T^{-1}-1$ | 1 | $-T^{-1}$ |

$$\frac{1}{\Delta} = Z_0$$

$$-\frac{T\partial_T\Delta}{\Delta^2} = \sum_{(s,i,j)\in D} \frac{s\langle(p_i - p_j)x_j\rangle}{\Delta}$$

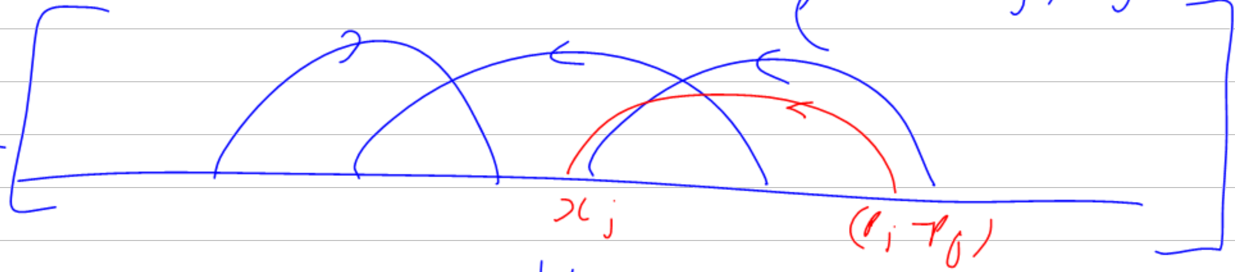
$x - p$

$\mathrm{Sym}(x) \sim \mathrm{Sym}(p)$



$\underline{p_i^2 x_i x_j}$

$$\partial_t \left[ e^{t(\rho_i - \rho_j) x_j} \right]$$

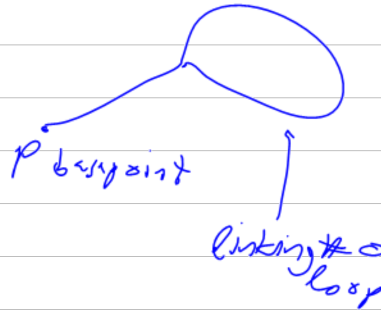




$$= \partial_t \frac{1}{\Delta(e^t)} = \frac{T \partial_T \Delta}{-\Delta^2}$$

K: knot, $C = K^c$, $\tilde{C}$ = univ $\mathbb{Z}$-cover

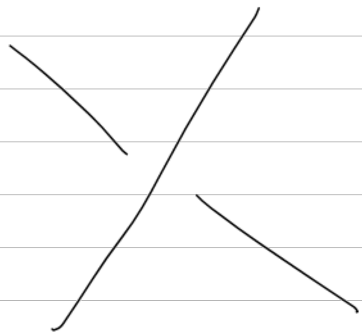
lasso:



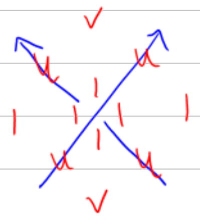

$H_1(\tilde{C}) = \langle \text{lassos} \rangle / ?$

What is the action of $\mathbb{Z}[T^{\pm 1}]$?

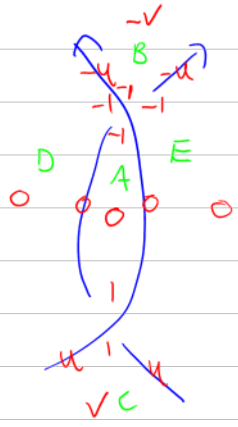
"$H_1(\tilde{C})$ is $\mathbb{Z}[T^{\pm 1}]$-torsion"?



# R2 invariance of Kashaev's signature




$V = 2u^2 - 1$

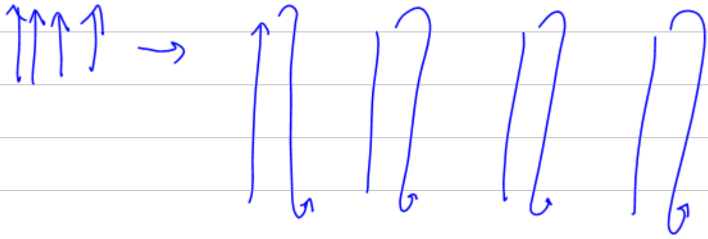


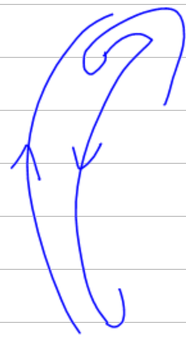

$$\left[\begin{array}{c|c|c} A & B & 0 \\ \hline B^T & \boxed{C+D} & E \\ \hline 0 & E^T & F \end{array}\right]$$

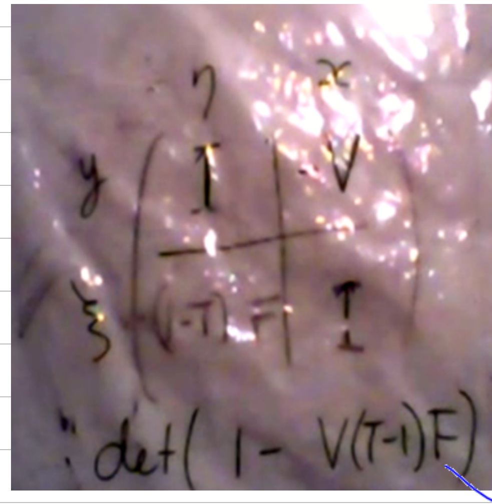
| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 0 | -1 | 1 | 0 | 0 |
| B | -1 | -v+.. | 0 | -u | -u |
| C | 1 | 0 | v+... | u | u |
| D | 0 | -u | u | 0 | 0 |
| E | 0 | -u | u | 0 | 0 |

Testing my new space desk software.

$$\int e^{-x^2/2} = \sqrt{2\pi}$$

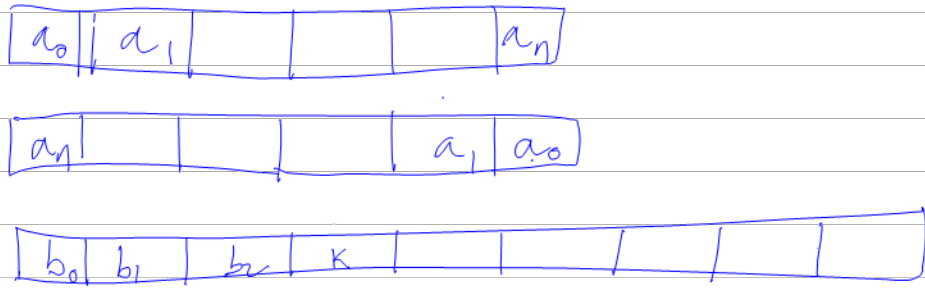







$F - V(T-1)$

$TV - V^T = TV - V - F$



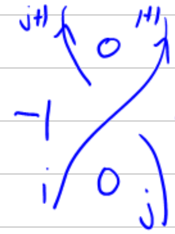

$b_0 = a_0 a_n$ $\quad$ $b_n = a_0$

$A(t) = f(t)f(t^{-1})$ $\quad$ $B(x) = A(x)A(-x)$

$$(b_0 \cancel{+ b_1 x} + b_2 x^2 + b_3 x^3 + \dots)$$

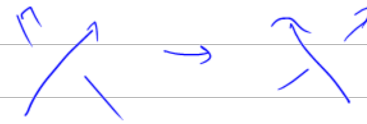
$$= (a_0 + a_1 x + a_2 x^2 + \dots)(a_0 - a_1 x + a_2 x^2 + \dots)$$

$$\begin{pmatrix} -T & T-1 \\ 0 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} -T^{-1} & T^{-1}-1 \\ 0 & -1 \end{pmatrix}$$



$$\begin{pmatrix} T^{-2}-T^{-1} & \\ ? & 1 \end{pmatrix}$$

$$\begin{array}{ccc} & i+1 & j+1 \\ & 1 & -T^{-1} \\ i & -T^{-1} & \begin{pmatrix} -T^{-1} & 1 \\ x & -T \end{pmatrix} \\ j & 1 & \end{array} \Big/ -T$$



$$\begin{pmatrix} 1 & 0 & -T & T-1 \\ 0 & 1 & 0 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} -T^{-1} & 0 & +1 & T^{-1}-1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & \\ & & & & 1 \\ \multicolumn{4}{c}{\longrightarrow \begin{pmatrix} -T^{-1} & T^{-1}-1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}} & \\ & & & & 0 \end{array} \to \begin{pmatrix} -T^{-1} & T-1 & -T & 0 \\ 0 & -1 & 0 & -T^{-1} \end{pmatrix}$$

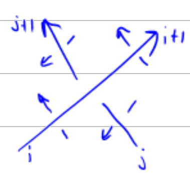
$\to \Big($

$$\alpha = \begin{pmatrix} \alpha & 0 \\ b & 1 \end{pmatrix}$$

$$(I \;\; A) \longmapsto (P \;\; PA) \longrightarrow \left(P\alpha, PA\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\alpha\right)$$

$$\to \left(I, \alpha^{-1} \cancel{P^{-1}} PA \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \alpha\right)$$

---



$\gamma_i = \gamma_{i+1}$

$\gamma_j \gamma_i \gamma_{j+1}^{-1} \gamma_{i+1}^{-1} = 1$

$$\begin{array}{ccccc} & i & j & i+1 & j+1 \\ i & 1 & 0 & -1 & 0 \\ j & 0 & 1 & T-1 & -T \end{array}$$

or, with inverses:

$\gamma_i^{-1} = \gamma_{i+1}^{-1}$

$\gamma_j^{-1} \gamma_i^{-1} \gamma_{j+1} \gamma_{i+1} = 1$

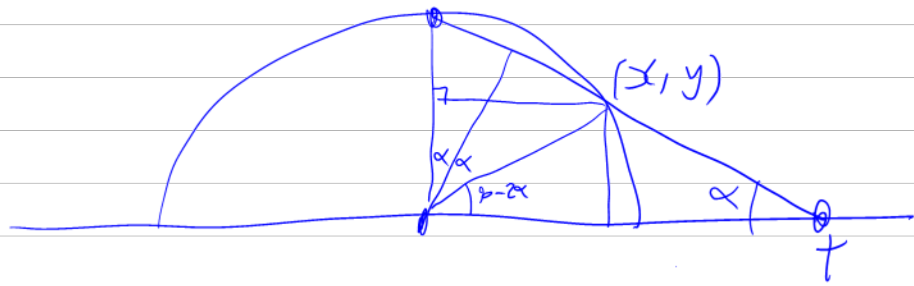
$$\begin{array}{ccccc} & i & j & i+1 & j+1 \\ i & 1 & 0 & -1 & 0 \\ j & & & & \end{array} \sim \begin{pmatrix} 1 & 0 & -1 & 0 \\ & & & \end{pmatrix}$$

$$T(1-T)+(1-T)^2=(1-T)$$

180515 **Q.** Wherefore the zip algebra, $\langle z_n\rangle_{n\geqslant 0}$ with $z_m z_n = \sum_{k=0}^{\min(m,n)} k!\binom{m}{k}\binom{n}{k} z_{m+n-k}$? Representation: $z_n \mapsto \hat{x}^n\partial_x^n$. Isomorphic to $\mathbb{Z}[y]$ via $z_n \mapsto y(y-1)\cdots(y-n+1)$.


$$F(P_2,P_3)$$
$$+F(P_1,P_3)$$
$$+F(TP_1+(1-T)P_3,\ TP_2+(1-T)P_3)$$

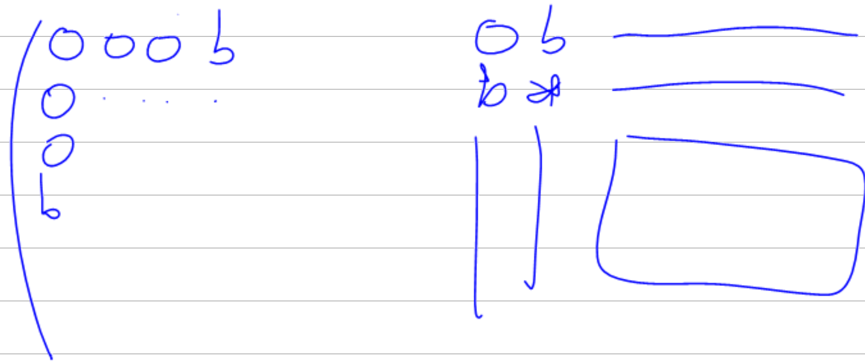

$$F(P_1,P_2)$$
$$+F(P_1,P_3)$$
$$+F(TP_2+(1-T)P_1,\ P_3)$$



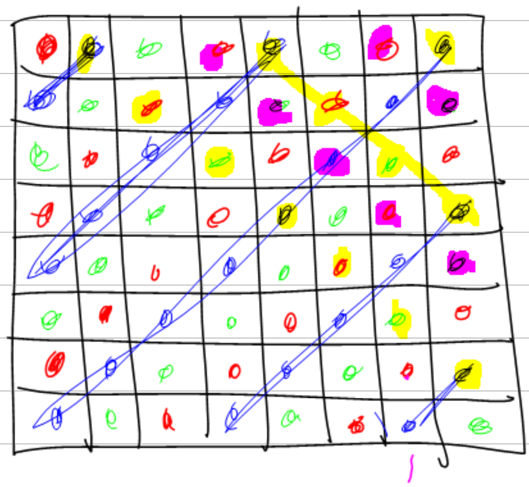

$t = \frac{x}{1-y}$

$y = 1 - x/t \qquad x^2 + \left(1 - \frac{x}{t}\right)^2 = 1 \qquad x^2 - \frac{2x}{t} + \frac{x^2}{t^2} = 0$

$\left(1 + \frac{1}{t^2}\right)x = \frac{2}{t} \qquad \frac{t^2+1}{t^2}x = \frac{2}{t} \qquad (t^2+1)x = 2t \qquad x = \frac{2t}{1+t^2} \qquad y = 1 - \frac{2}{1+t^2} = \frac{t^2-1}{1+t^2}$



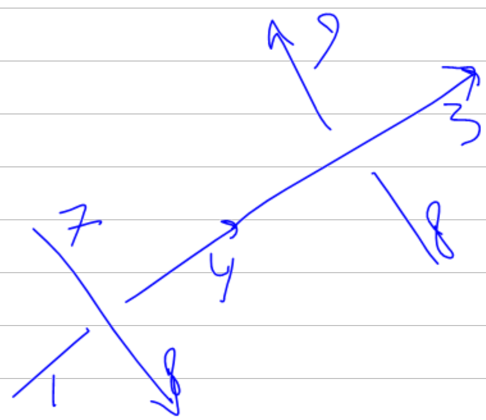
$? \circlearrowright \frac{\pi}{[\pi,\pi]} \xrightarrow{\text{Fix!}} \mathbb{Z} \circlearrowleft \cdot(-1)$

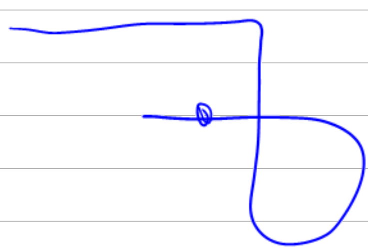


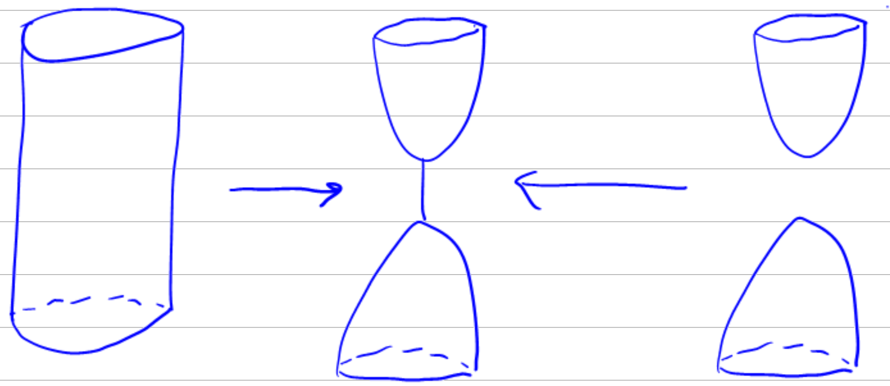
21

22

21





$$\tilde{P} \longrightarrow \tilde{C} \longrightarrow \tilde{C}/\tilde{P}$$

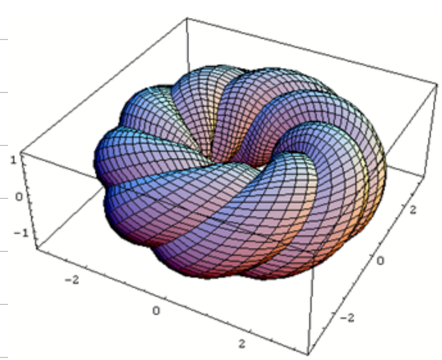
$$0 = H_1(\tilde{P}) \longrightarrow H_1(\tilde{C}) \longrightarrow H_1(\tilde{C}, \tilde{P}) \longrightarrow H_0(\tilde{P}) \longrightarrow H_0(\tilde{C})$$

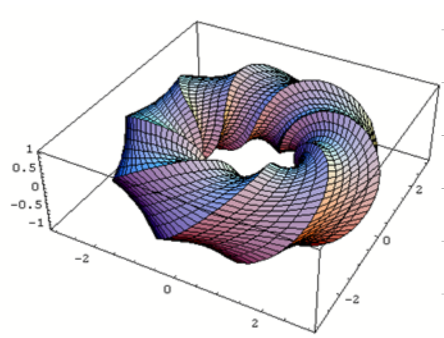
$$H_0(\tilde{P}) = \mathbb{R}, \qquad H_0(\tilde{C}) = \mathbb{Z}$$

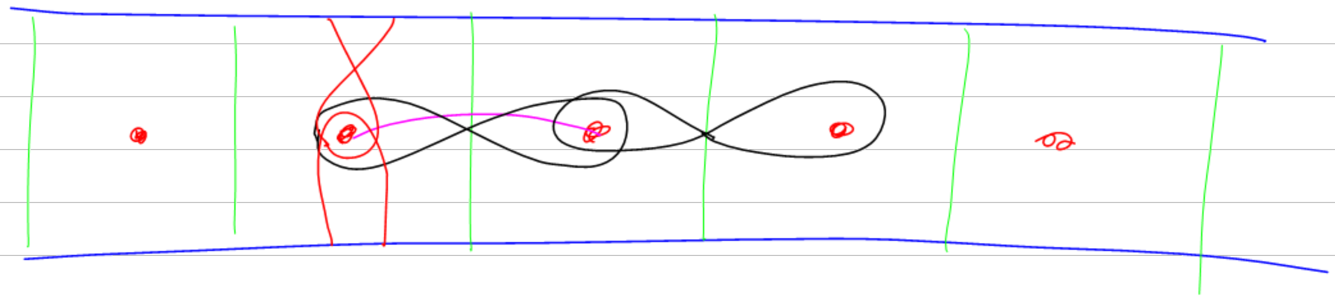
$$0 \longrightarrow CA(H) \longrightarrow H_1(\tilde{\Sigma}, \tilde{P}) \longrightarrow H_1(\tilde{C}, \tilde{P}) \longrightarrow 0$$

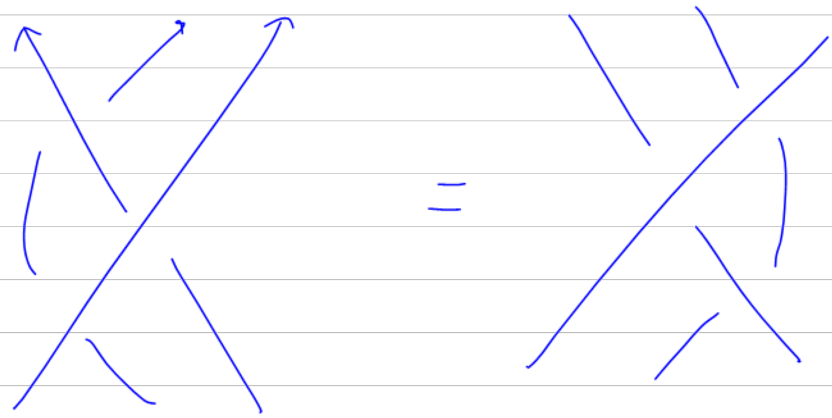
$$A(H) = H_1(\tilde{C}) =$$

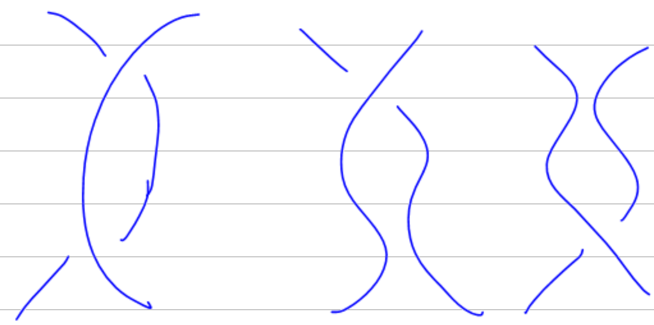
$$A(H) \oplus \mathbb{R}/\mathbb{Z} \cong \Omega / CA(H)$$





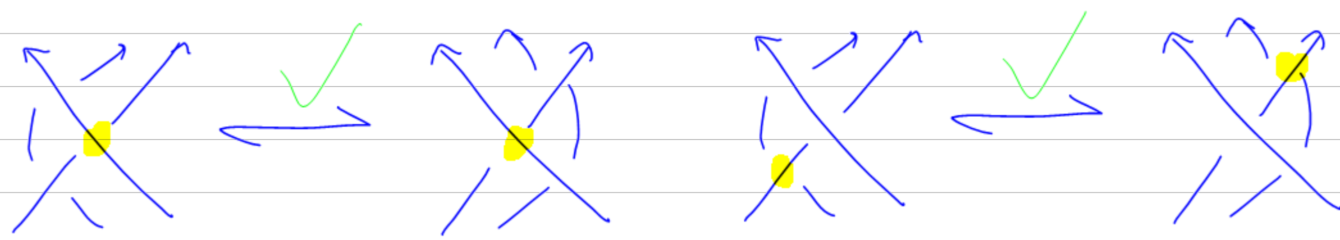


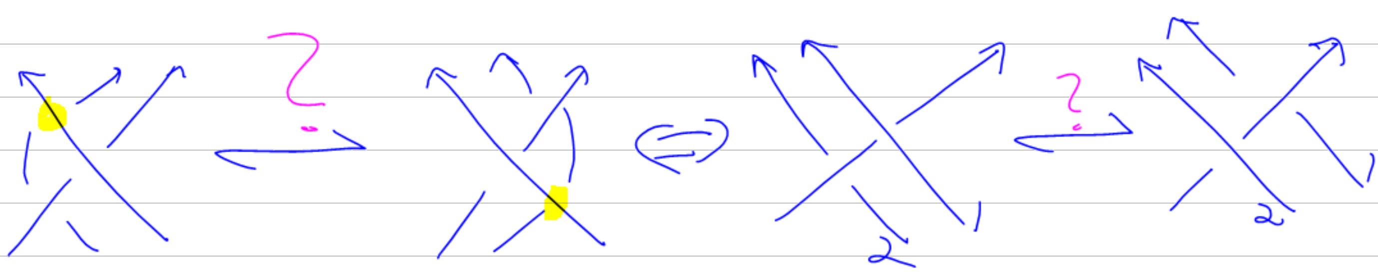


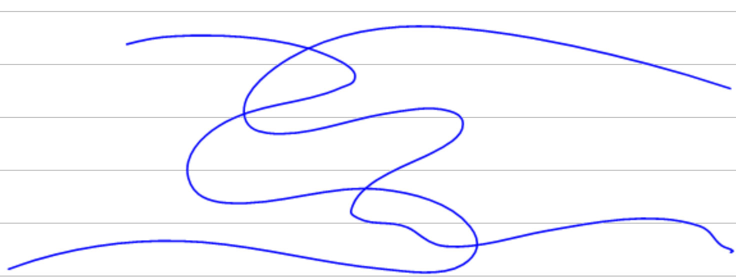
**Thm?** $\phi(K) := \sum_{\substack{C \in K \\ \text{type } \curvearrowright}} s(C)\, x^{s(C)+lk(K/C \to ?(?))}$

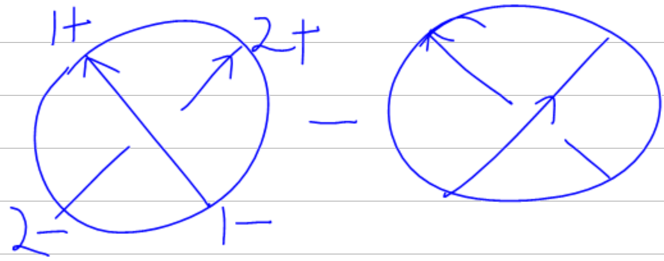
is an invariant of long knots.

Pf



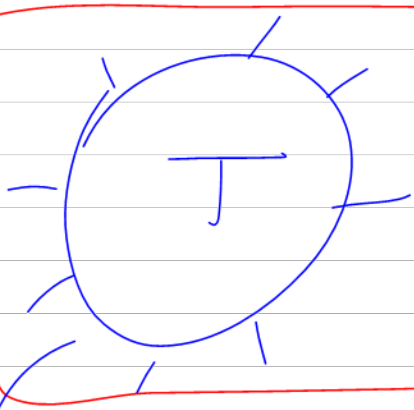






$= -2$





$\longrightarrow$ $Fi_1$ of all completions.

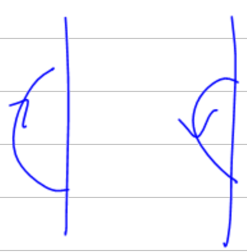
Formalize & implement!

{Skeleton (including bdry specs)} ☐

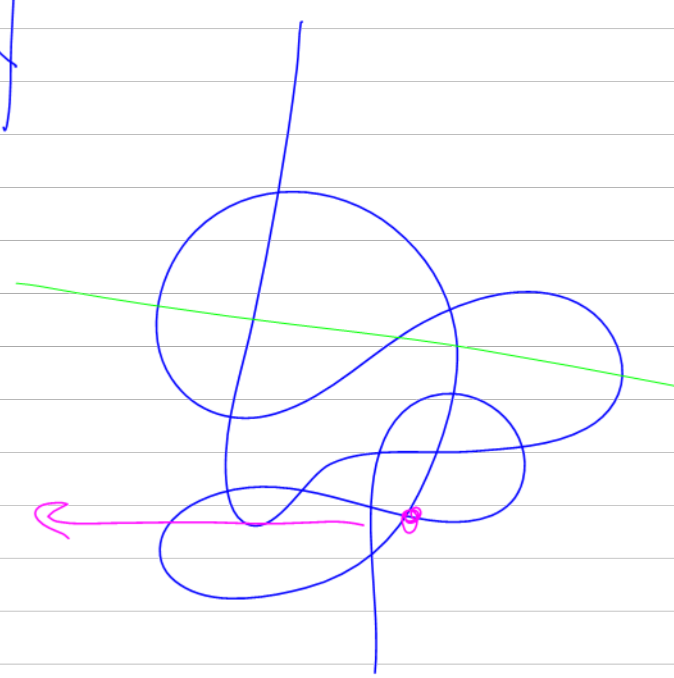
{linking matrix} ☐

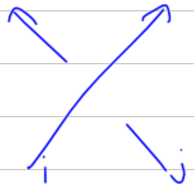
(~~for each crossing type~~), a functional

{closure types} $\longrightarrow$ linking functionals of complement. [affine info?]



$x^{\sum l_{ij}}$

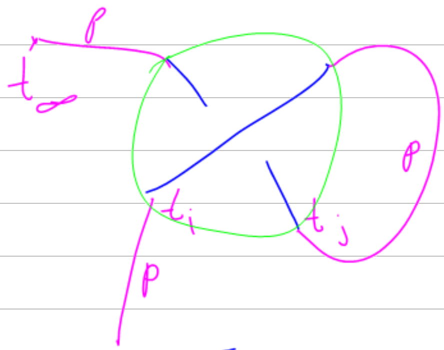





$(t_i t_j h_i h_j)$

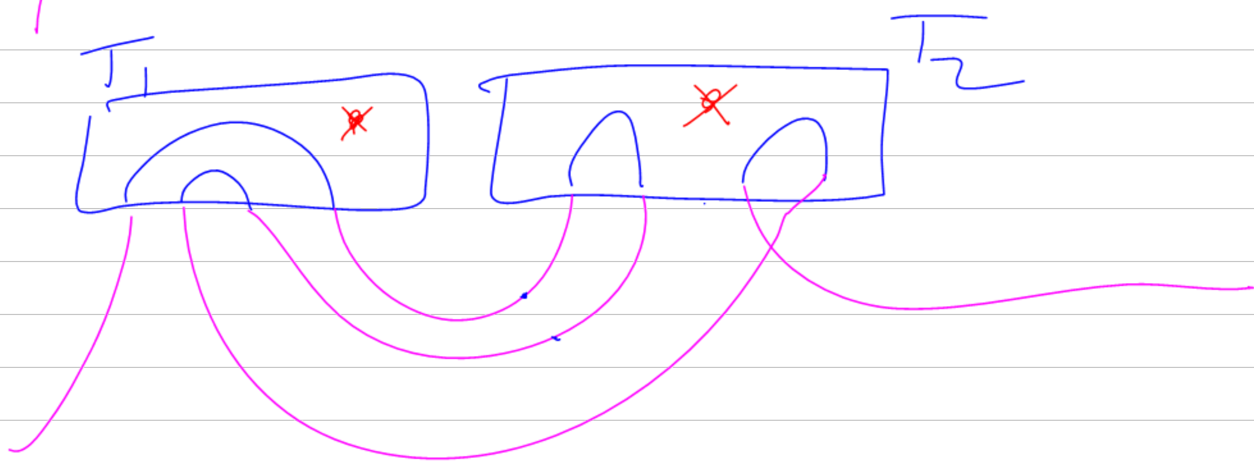
$b_i n_j$

$x^7 \leftrightarrow x[7]$




$$(t_i t_j t_\infty) \longmapsto x\Big[\lambda + \sum_{\alpha,\beta \in \{i,j,\infty\}} c_{\alpha\beta} l_{\alpha\beta}\Big]$$

$$(t_j t_i t_\infty) \longmapsto 0$$





$$\longrightarrow \sum x\Big[\sum_{1,2,\infty} l_{\alpha\beta}\Big]$$

$l_{21} \longrightarrow$