$$\frac{1}{(1-a)(1-c)-b^2} = \frac{1}{1-c}\,\frac{1}{1-\left(a+\frac{b^2}{1-c}\right)}$$

$$\frac{1-d}{(1-a)(1-d)-bc} = \frac{1-d}{(1-d)\left(1-\left(a+\frac{bc}{1-d}\right)\right)}$$

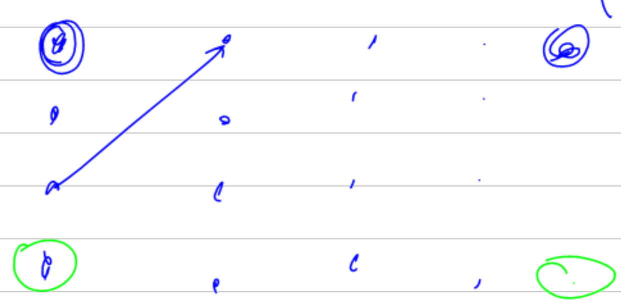
$y\,b\,a\,x \rightarrow b\,y\,x\,a$

$1\,0\,2\,1 \qquad \underbrace{0\,\overset{\text{inner}}{1\,1}\,2}_{\text{outer}}$

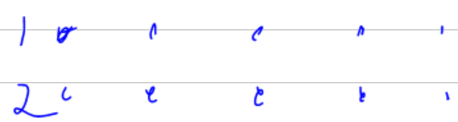
$a\,b\,y\,x \qquad a\,x\,b\,y$

$t = b \pm \epsilon a$

$$(I-A)^{-1} = \sum A^k = E = \begin{pmatrix} e_{11} & & \\ & \ddots & \\ & & \ddots \end{pmatrix}$$



$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad e_{11} = \frac{1-d}{(1-d)(1-a)-bc}$$

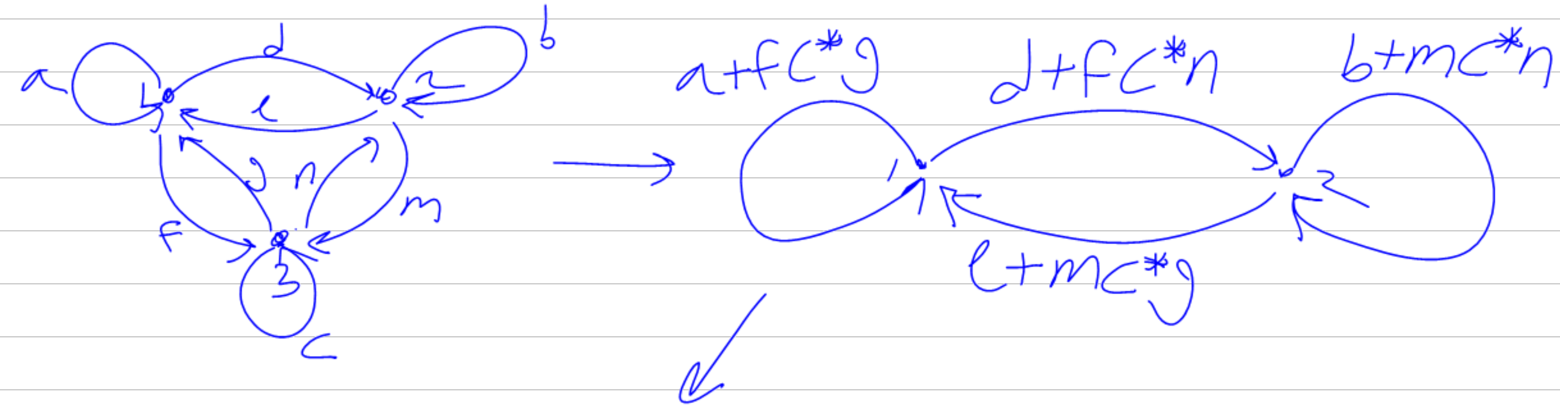


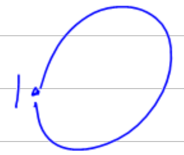
12112121

$$1\,1^*(2\,1^*)^*$$

$$\downarrow$$

$$\frac{1}{1-a}$$





$a+fc^*g+(d+fc^*n)(b+mc^*n)^*(e+mc^*g)$

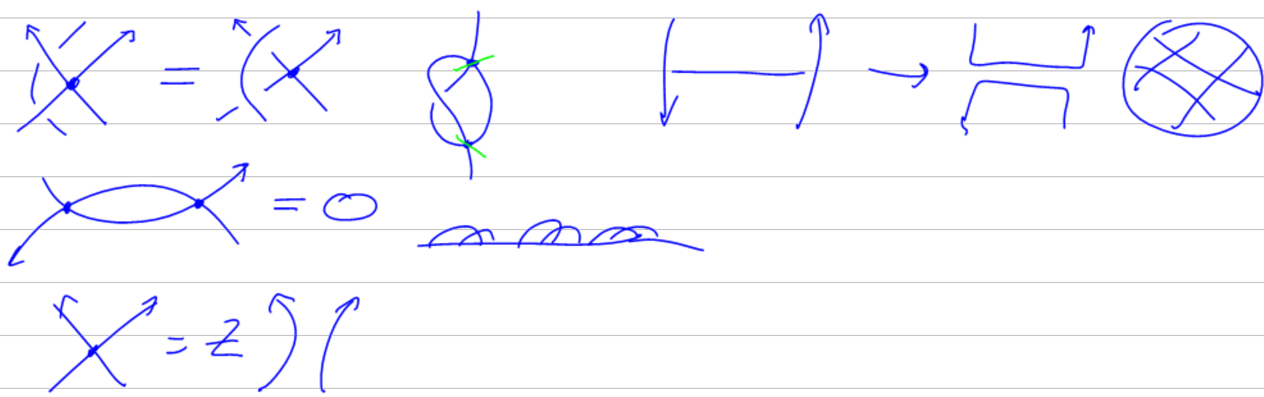
$$\text{Simplify}\left[a+f\frac{1}{1-c}g+\left(d+f\frac{1}{1-c}n\right)\frac{1}{\underbrace{1-\left(b+m\frac{1}{1-c}n\right)}}\left(e+m\frac{1}{1-c}g\right)==a+\frac{(1-c)\,d\,e+(1-b)\,f\,g+d\,g\,m+e\,f\,n}{(1-b)\,(1-c)-m\,n}\right]$$

True

$$\frac{1-c}{1-(b+c-bc+mn)}=\frac{1-c}{(1-b)(1-c)-mn}$$

$$\| $$

$$a+(db^*e+fc^*g+db^*mc^*g+fc^*nb^*e)\cdot(mb^*nc^*)^*$$



$$\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\beta,\ T_a T_b \to T_c]{m^{ab}_c} \begin{array}{c|cc} \mu\omega & c & S \\ \hline c & \gamma+\alpha\delta/\mu & \epsilon+\delta\theta/\mu \\ S & \phi+\alpha\psi/\mu & \Xi+\psi\theta/\mu \end{array} \qquad \begin{array}{c|cc} \omega & c & S \\ \hline c & \alpha & \theta \\ S & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\alpha]{\Gamma::tr_c} \begin{array}{c|c} \mu\omega & S \\ \hline S & \Xi+\psi\theta/\mu \end{array}$$

$\Xi+\psi\theta+\alpha\Xi$

210211a Halacheva: $\mathcal{A}(X) := \bigoplus_k \mathrm{End}(\Lambda^k X)$ is a traced meta monoid with $m^{xy}_z(A) := (z \to y)/\!/(e_x/\!/i_x/\!/A/\!/e_y/\!/i_y - e_x/\!/A/\!/i_y)/\!/(\cancel{x} \to z)$ and $\mathrm{tr}_x(A) := e_x/\!/i_x/\!/A/\!/e_x/\!/i_x - e_x/\!/A/\!/i_x$. Contains Γ (w/ fixed colours) via Υ: $(\omega, M) \mapsto \omega\Lambda^*(M)$. Predict $\mathcal{A}$ from Γ? Interpret $\mathcal{A}$ in $ybax$? Related to super-algebras? Raise $\mathcal{A}$ to meta-Hopf? Understand $\mathrm{im}(\Upsilon)$?

z

$\boxed{\text{LTR matrix conventions}}$

$$\begin{array}{c} \\ x \\ y \\ r \end{array}\!\!\begin{array}{c} \begin{array}{ccc} x & y & r \end{array} \\ \begin{pmatrix} \alpha & \beta & \theta \\ \gamma & \delta & \epsilon \\ \phi & \psi & \Xi \end{pmatrix} \end{array} \xrightarrow{(z\to y)/\!/e_x/\!/i_x/\!/A/\!/e_y/\!/i_y/\!/x\to z} \begin{array}{c} \\ z \\ r \end{array}\!\!\begin{array}{c} \begin{array}{cc} z & r \end{array} \\ \begin{pmatrix} \gamma & \epsilon \\ \phi & \Xi \end{pmatrix} \end{array}$$

$$-(z\to y)/\!/e_x/\!/A/\!/i_y/\!/x\to z \quad \begin{array}{c} \\ z \\ r \end{array}\!\!\begin{array}{c} \begin{array}{cc} z & r \end{array} \\ \begin{pmatrix} +\beta\delta & \\ & \end{pmatrix} \end{array}$$

deg	0	1	2
w	w	wM	wM^2

$$w \longmapsto w - \beta$$

$$\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\beta,\ T_a T_b \to T_c]{m^{ab}_c} \begin{array}{c|cc} \mu\omega & c & S \\ \hline c & \gamma+\alpha\delta/\mu & \epsilon+\delta\theta/\mu \\ S & \phi+\alpha\psi/\mu & \Xi+\psi\theta/\mu \end{array} \qquad \begin{array}{c|cc} \omega & c & S \\ \hline c & \alpha & \theta \\ S & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\alpha]{\Gamma::tr_c} \begin{array}{c|c} \mu\omega & S \\ \hline S & \Xi+\psi\theta/\mu \end{array}$$

$$\mu w \cdot (\gamma + \alpha\delta/\mu) = w(\mu\gamma + \alpha\delta)$$
$$= w\left(\left(1-\frac{\beta'}{w}\right)\frac{\gamma'}{w} + \frac{\alpha'\delta'}{w^2}\right) =$$
$$= \frac{1}{w}\left((w-\beta')\gamma' + \alpha'\delta'\right)$$

$$e^{\alpha\xi x + \phi\xi y + \theta\eta x + \Xi\eta y}/\!/(1+x\xi) = e^{\Xi\eta y}\left(1+(\alpha+\phi\theta y\eta)\right)$$
$$= (1+\alpha)\left(1+\frac{\phi\theta y\eta}{1+\alpha}\right)e^{\Xi\eta y} = (1+\alpha)\,e^{\left(\Xi+\frac{\phi\theta}{1+\alpha}\right)\eta y}$$

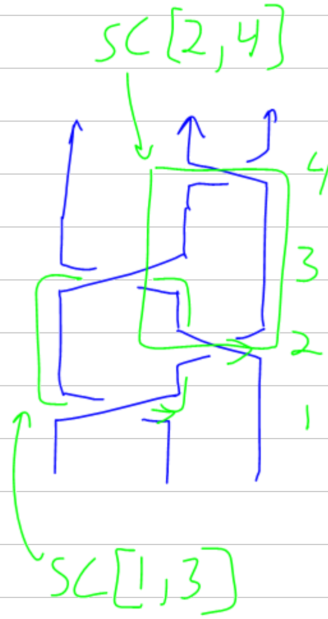
$$e^{\sum a_i x_i} = \prod (1 + a_i x_i)$$

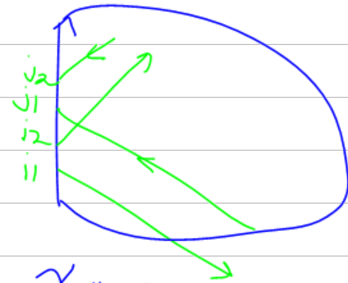
For bosons only!

$$\begin{tikzcd}
S(V) \arrow[dr, "\mathrm{lin}"] & \\
V \arrow[u, "\exp"] \arrow[r, "\text{non-lin}"'] & W
\end{tikzcd}$$

when does it factor through $\sim \Lambda$?

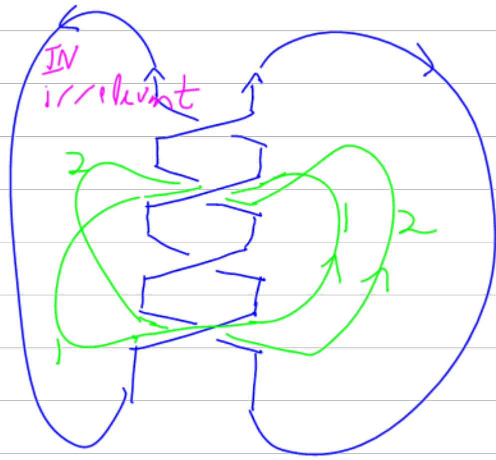
Tristram-Levine Signatures

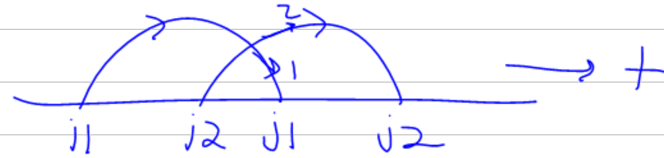




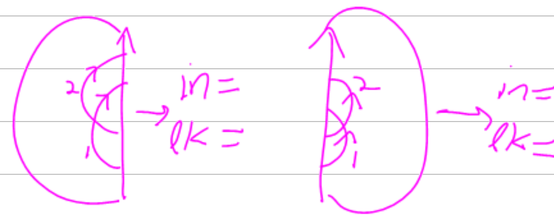
$x_{i1>i2}$

$$lk(a,b^+) = \sum_{x: z(a)>z(b)} \operatorname{sign}(x)$$

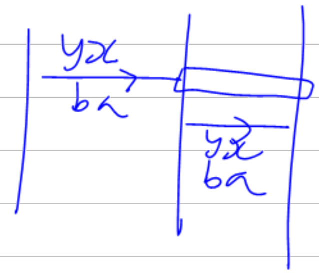




$IN(i1 \to j1,\ i2 \to j2) = \ldots$



$[a, x] = x \qquad x^2 = 0$

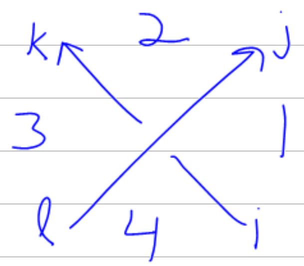


ybx

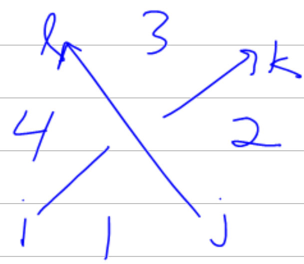
ybx

The signature program at

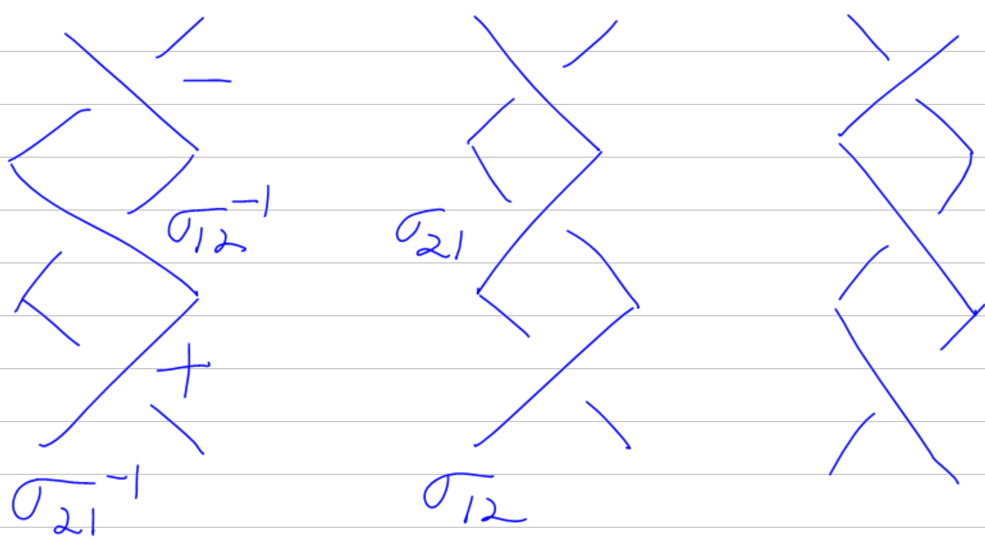
AP/Projects/KnotTheory/Testing/Testing Knot Signature.nb:

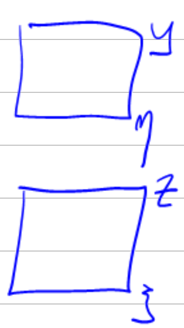


X_p



X_m





bad: ηz

$$\begin{tikzcd}
u\mathcal{D} \arrow[r, hook] \arrow[d, "P_u"] & v\mathcal{D} \arrow[r, shift left, "s"'] \arrow[dl, "P_v"] & v\mathcal{D} \arrow[l, shift left, "s^{-1}"'] \arrow[dl, "w"] & \supset v\mathcal{D}_{>n} \\
\langle K_D \rangle \arrow[r, "\pi"] & uK/uK_{n+1} & &
\end{tikzcd}$$

vanishes if $\#\times + \#\times > n$

"Canonical form diagrams, CF"

P_v: 1. Brush xings off the maximal tree for the $\times$.
2. Flip the 1st non-dec xing, paying w/ $\times$.
3. Realize ("v-xings") as descending.

iterate, D!

NTS: $w = s^{-1} // P_v // \pi$ vanishes on $v\mathcal{D}_{>n}$

use downward induction on $\#\times$.

Step 1 w vanishes on CF/VCF diags w/ $\#\times > 0$.

Step 2 w vanishes on clear-tree diagrams with a non-descending xing.

Step 3 The rest.

Proof of GPV (2)

$$\begin{array}{ccccccc}
uD & \hookrightarrow & vD & \underset{s}{\overset{s^{-1}}{\rightleftarrows}} & vD & \supset & vD_{>n} \\
\downarrow P_u & \swarrow P_v & & \swarrow w & & & \\
uK & & \xrightarrow{\pi} & uK/uK_{n+1} & & &
\end{array}$$

vanishes if $\#\not{X} + \#X' > n$

Goal: Construct $\qquad \not{X} := X_{\prime} - X$

$$P : vD\langle \not{X}, X' \rangle \longrightarrow uD\langle \not{X}, X_{\prime} \rangle / \#\not{X} > n$$

s.t. 1. $P|_{uD} = Id$ mod R-moves & $\not{X} = X_{\prime} - X'$

2. $P|_{\#\not{X} + \#\not{X} > n} = 0$

if $\#\not{X} > n$ in D $\quad P(D) = 0$; Suppose P is defined & satisfies 1 & 2 if $\#\not{X} > K$, and D has $\#\not{X} = K$.

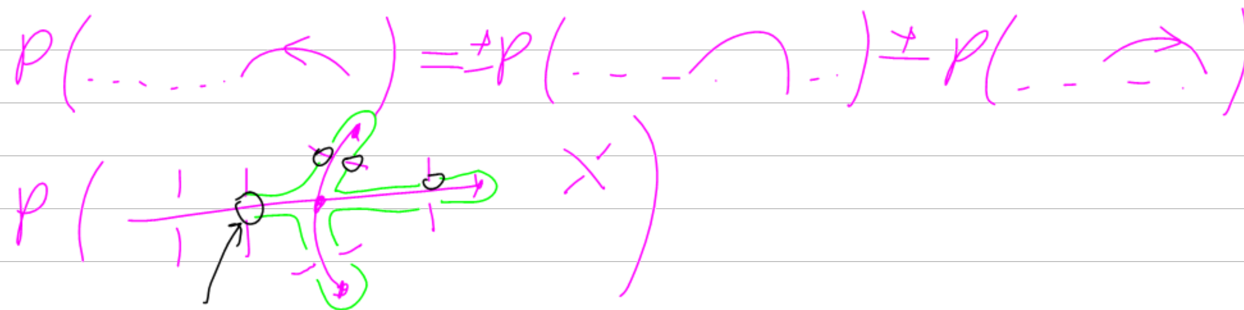
* For every D choose a tree $T(D)$ (for X), indep of $X_{\prime}$.

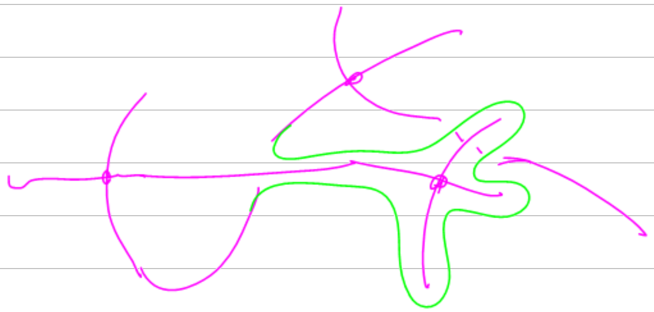
* Aside A "good choice function" on $S(X)$, the set of non-empty subsets of a finite set X, is $c : S(X) \to X$ s.t. 1. $c(A) \in A$

2. If $b \in A \setminus \{c(A)\}$ then $\underline{c(A \setminus \{b\}) = c(A)}$.

Thm {good choice fnctns} $\longleftrightarrow$ {total orderings of X}

* Define P by fixing the first issue.





$$\left[\begin{matrix} ba + yx \\ + ab + xy \end{matrix},\ x_1 - x_2\right] \overset{?}{=} 0$$

$$\left[\begin{matrix} ba + yx \\ + ab + xy \end{matrix},\ y_1 - y_2\right] = by \mp yb$$

$$0 = \partial_\epsilon (Q Q^{-1}) = (\partial_\epsilon Q) Q^{-1} + Q (\partial_\epsilon Q^{-1})$$

$$\Rightarrow \partial_\epsilon Q^{-1} = Q^{-1} (\partial_\epsilon Q) Q^{-1}$$

$$\lambda^{ij} a_i b_j + \frac{1}{2} q^{ij}(b) x_i x_j + P(a, x)$$

$$\lambda^{ij} a_i b_j + \frac{1}{2} q^{ij}(b) x_i x_j + a_i \alpha^i + b_i \beta^i + x_i \xi^i$$

$$[a, \bar{b}]$$

$$(b, a) = [b, \bar{a}] = -[\bar{a}, b]$$

$$= -\overline{[a, \bar{b}]}$$

$$= -\overline{(a, b)}$$

$$\int e^{i\lambda x^2/2}\,dx = \frac{\sqrt{2\pi}}{\sqrt{i\lambda}} \qquad \log(1-x) = -\sum_{k\geq 1} \frac{x^k}{k}$$

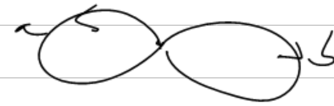
$$\sum (1-i\lambda)^k = \frac{1}{1-(1-i\lambda)} = \frac{1}{i\lambda}$$

$$\sum \frac{(1-i\lambda)^k}{2k} = -\frac{1}{2}\log(i\lambda)$$

$$(z-\bar{z})\otimes(z+\bar{z}) = zz - \bar{z}\bar{z} + z\bar{z} - \bar{z}z$$

$$[a,z] = \bar{z} \qquad [a,\bar{z}] = z \qquad [z,\bar{z}] = 1$$

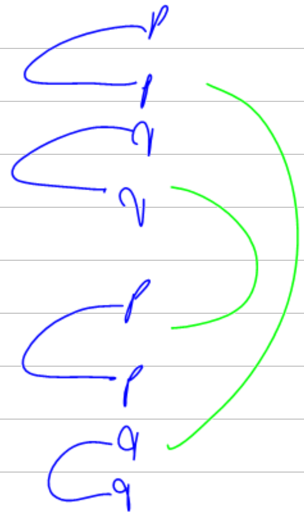
$$\frac{1}{1-a-b\frac{1}{1-d}c}$$



$$(a+bd^*c)^* = a^*(bd^*ca^*)^*$$

$$= \frac{1}{1-a}\,\frac{1}{1-b\frac{1}{1-d}c\frac{1}{1-a}}$$

$$=$$



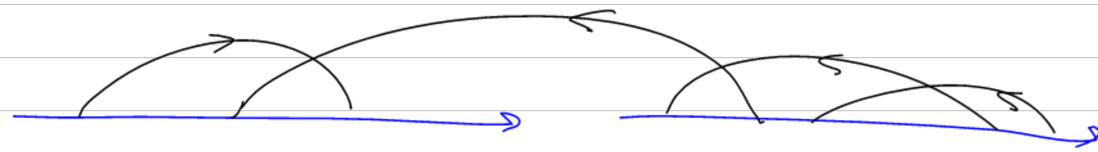
$$e^{\xi_1 x + \pi_1 p}\, e^{\xi_2 x + \pi_2 p}$$

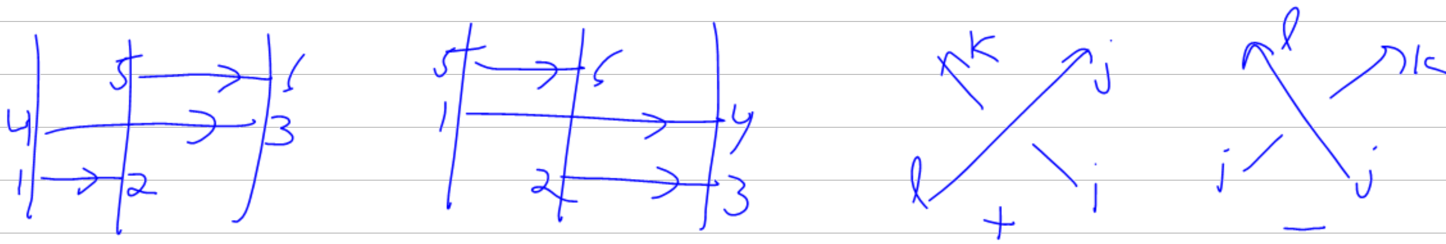
$$= e$$

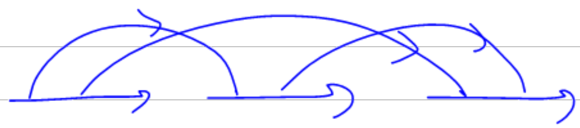
$$e^{\xi x} \int da\, db\, db'\, d\beta'\, e^{-b'\beta' + q(b')\frac{x^2}{2} + \lambda ab + \alpha a + (\beta+\beta')b}$$

Q. Can I always take an a for two x's?

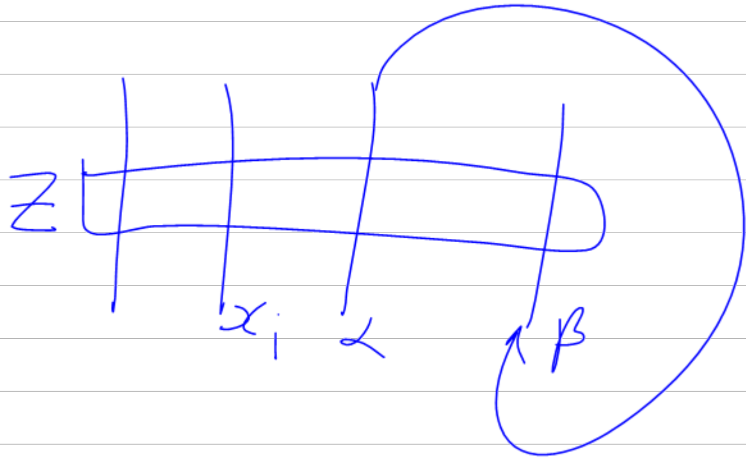
Thm: $MM(\mathcal{A}) \cong MM(gD)$

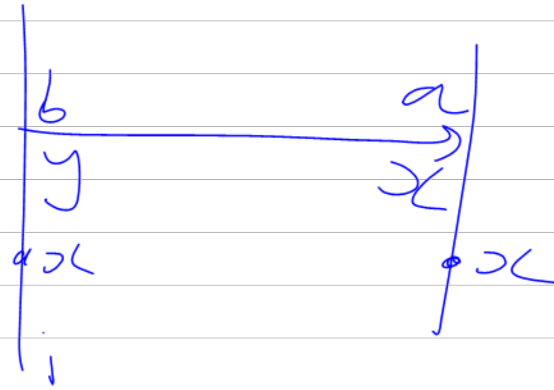






$$(\alpha,\beta)(\gamma,\delta) = (\alpha\beta\gamma\beta^{-1}, \beta\delta)$$

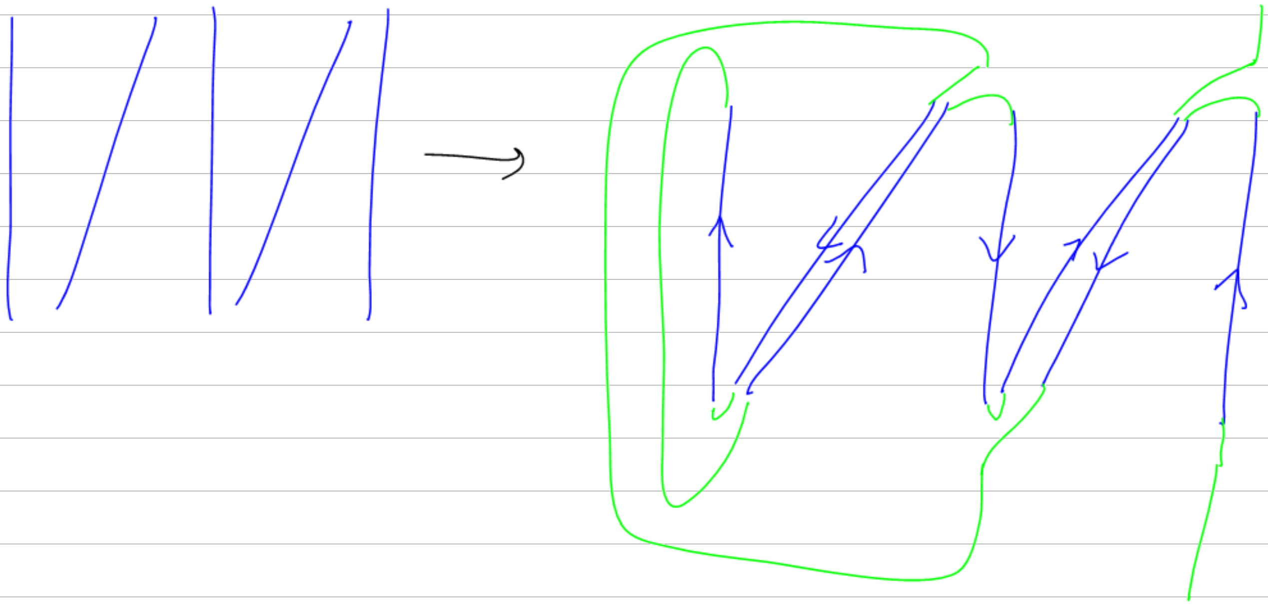


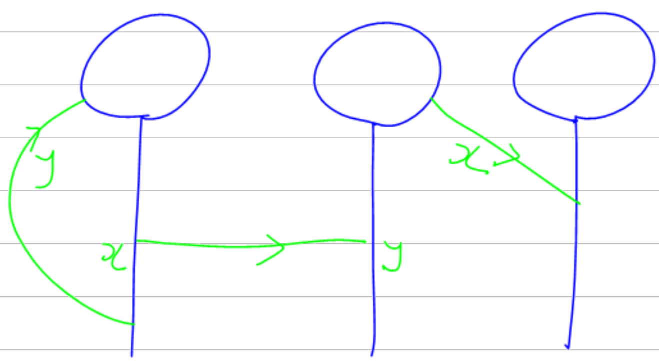


$$Z x_\alpha = x_\beta Z$$

$$x_i Z = \sum A_{ij} Z x_j$$

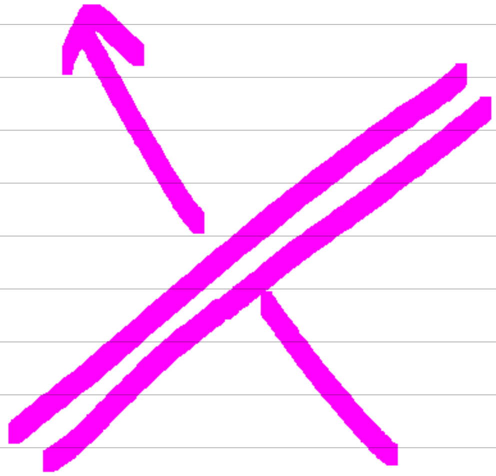
$$\begin{tikzcd}
\{Z\} \arrow[r] \arrow[r, bend left, "x\text{-scattering}"] \arrow[d, "m^{ij}_k"] & \{(W, A)\} \arrow[d, "\Gamma m"] \\
\{Z\} \arrow[r] & \{A\}
\end{tikzcd}$$



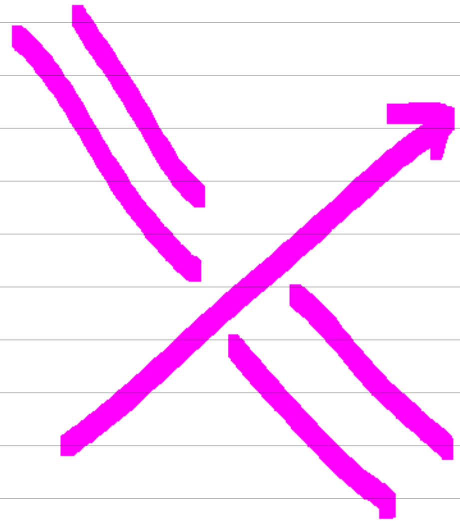

y
x
y
x

Goal: $\text{Exp}/m\left(c_0 + c_1 x + c_2 y + c_3 a + c_4 xy\right)$

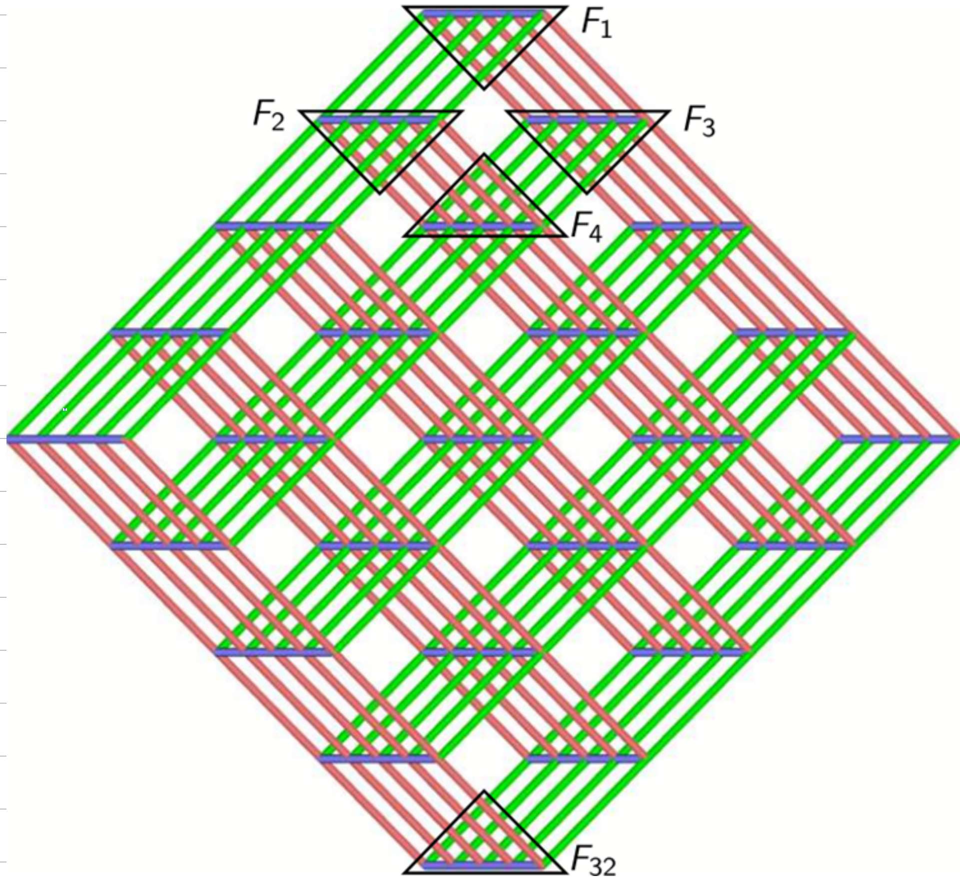
$$\log(Ab) = \log A + \log b$$



$\Delta \otimes 1$



$1 \otimes \Delta$



rectangle- line-
shark, feather,
multi-shark
multi-feather

The multi-feather method w/ multi-depth $\bar{b}$:

$$\left(\prod \frac{L^4}{2^{b_i}}\right)^{3/4} = L^{3d} / 2^{3/4\, b} \qquad b := |\bar{b}|$$

The multi-shark method w/ multi-depth :

$$\prod L^2 2^{b_i} = L^{2d} 2^{b} \quad = \quad = L^{\frac{18}{7} d}$$

$$L^{2d} 2^{b} = L^{3d} 2^{-\frac{3}{4} b}$$

$$2^{\frac{7}{4} b} = L^{d}$$

$$2^{b} = L^{4/7\, d}$$

$$L^{2d} 2^{b} = \left(L^{4d} 2^{-b}\right)^{2/3}$$

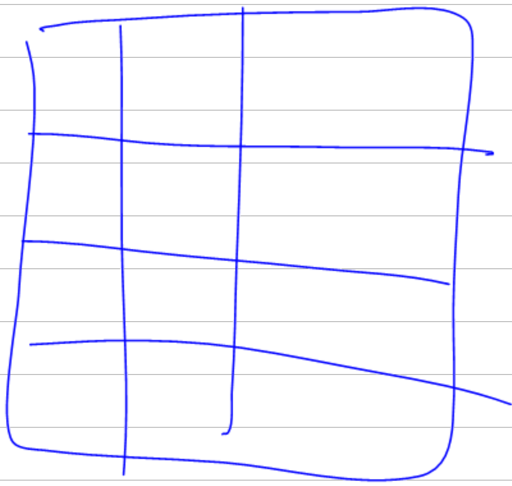
$$2^{\frac{5}{3} b} = L^{\frac{2}{3} b}$$

$$2^{b} = L^{\frac{2}{5} b}$$

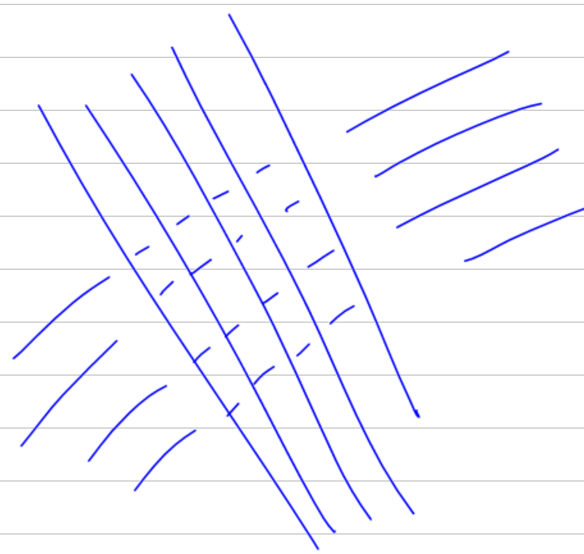
$$L^{\frac{12}{5} d} = V^{\frac{4}{5} d}$$

Hello Louis...

Hello Louis

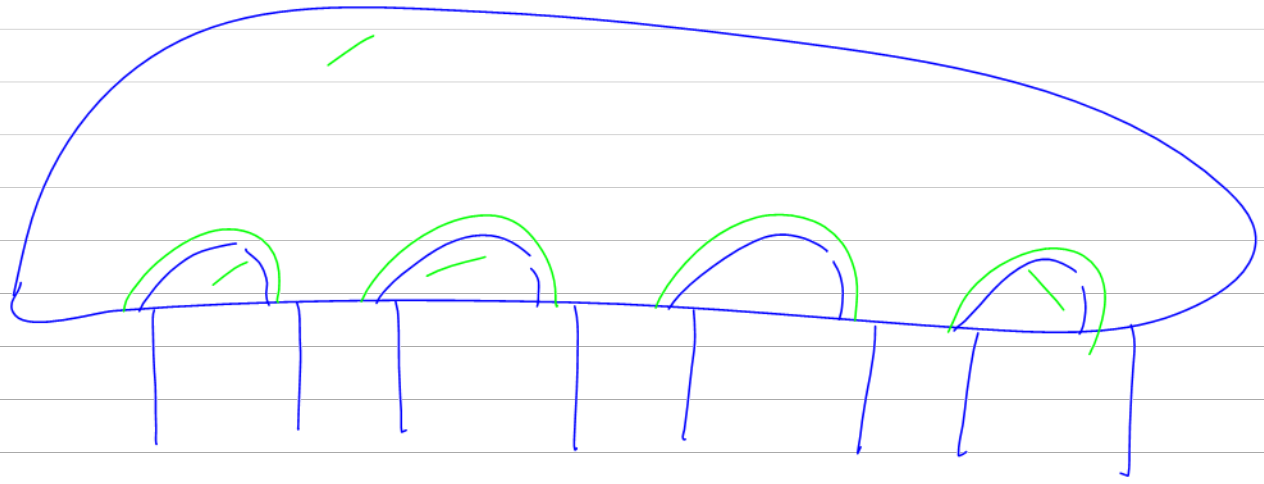


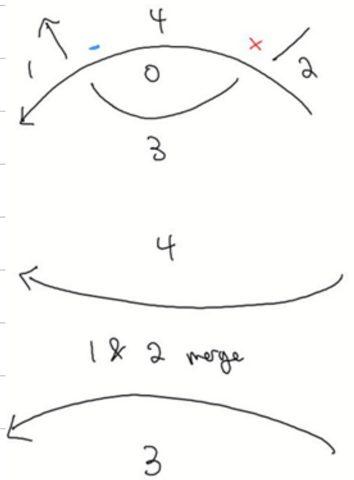
$V = L^3 \quad n = V^{4/3}$



$n^d \gg V^d$

$n^{\frac{2}{3}d} \gg V^{\frac{6}{7}d}$



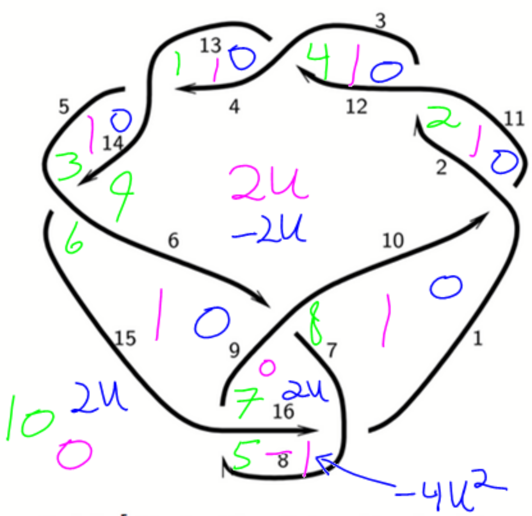


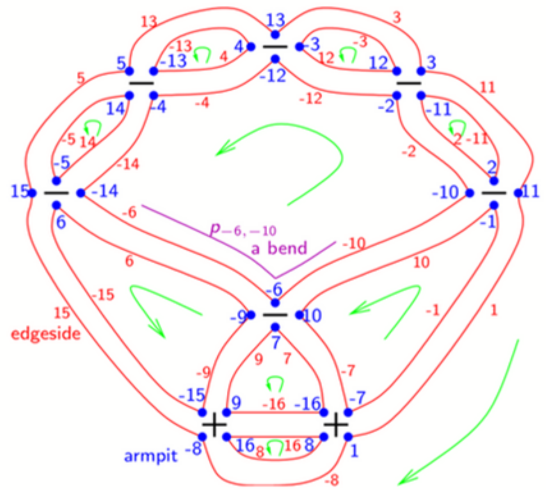
$$\begin{array}{c|ccccc}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & -1 & 1 & 0 & 0 \\
1 & -1 & x+1-2x^2 & b+0 & -x & -x \\
2 & 1 & 0 & 2x^2-1 & x & x \\
3 & 0 & -x & x & 0 & 0 \\
4 & 0 & -x & x & 0 & 0
\end{array}$$

row 2 = row 2 + row 1
col 2 = col 2 + col 1

$$\begin{array}{ccccc}
0 & -1 & 0 & 0 & 0 \\
-1 & 1-2x^2 & 1-2x^2 & -x & -x \\
0 & 1-2x^2 & 0 & 0 & 0 \\
0 & -x & 0 & 0 & 0 \\
0 & -x & 0 & 0 & 0
\end{array}$$

In[27]:=
```
PD[8₂] = PD[X[10, 1, 11, 2], X[2, 11, 3, 12], X[12, 3, 13, 4], X[4, 13, 5, 14], X[14, 5, 15, 6],
   X[8, 16, 9, 15], X[16, 8, 1, 7], X[6, 9, 7, 10]];
```





In[28]:=
```
Module[{K = 8₂, XingsByArmpits, bends},
  XingsByArmpits =
   List @@ PD[K] /. x : X[i_, j_, k_, l_] ⧴ If[PositiveQ[x], X₊[-i, j, k, -l], X₋[-j, k, l, -i]];
  bends = Times @@ XingsByArmpits /. _[X][a_, b_, c_, d_] ⧴ p_{a,-d} p_{b,-a} p_{c,-b} p_{d,-c};
  bends //. p_{x__,y_} p_{y_,z__} ⧴ p_{x,y,z}]
```

Out[28]= $p_{-13,4,-13}\, p_{-11,2,-11}\, p_{-5,14,-5}\, p_{-3,12,-3}\, p_{8,16,8}\, p_{6,-15,-9,6}\, p_{9,-16,7,9}\, p_{10,-7,-1,10}\, p_{-10,-2,-12,-4,-14,-6,-10}\, p_{1,-8,15,5,13,3,11,1}$

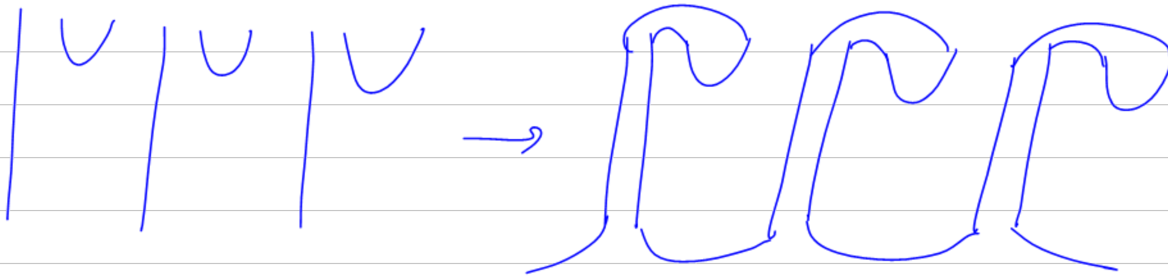
1 2 3 4 5 6 7 8 9 10

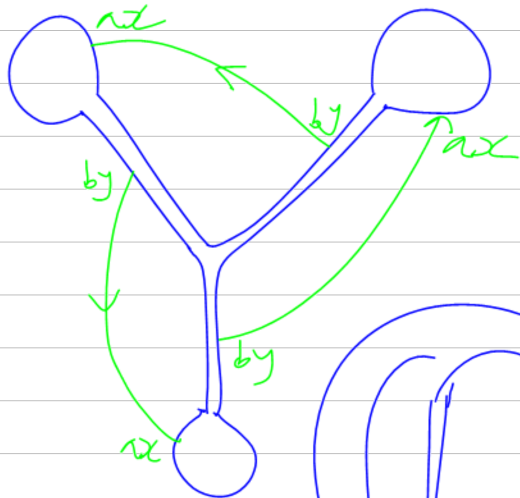
In[34]:=
```
2 u NullSpace[Kas[8₂, u]] // MatrixForm
```

Out[34]//MatrixForm=

$$\begin{pmatrix} -1 & -1 & -1 & -1 & 1-4u^2 & -1 & 2u & -1 & 0 & 2u \\ -1 & -1 & -1 & -1 & 1 & -1 & 0 & -1 & 2u & 0 \end{pmatrix}$$

1 2 3 4 5 6 7 8 9 10



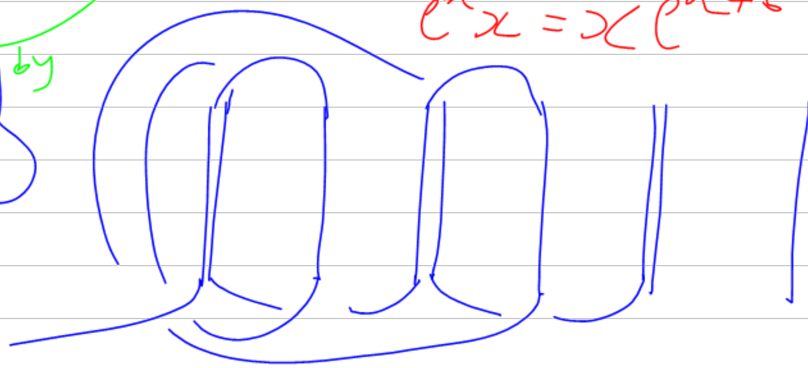


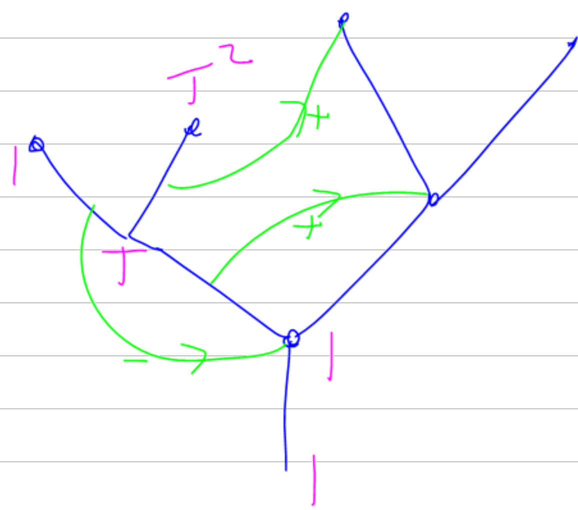
$ax - xa = x$

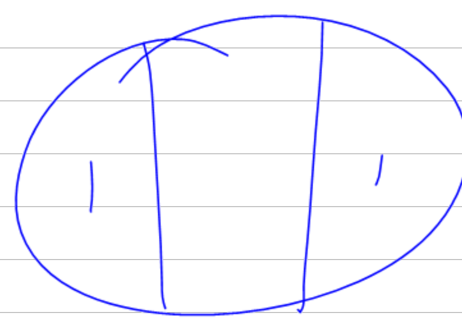
$ax = x(a+1)$

$a^n x = x(a+1)^n$

$e^{ba} x = x e^{a+b}$





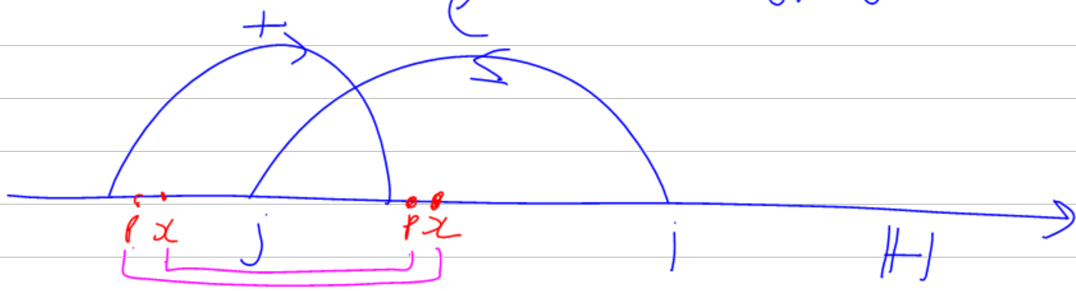


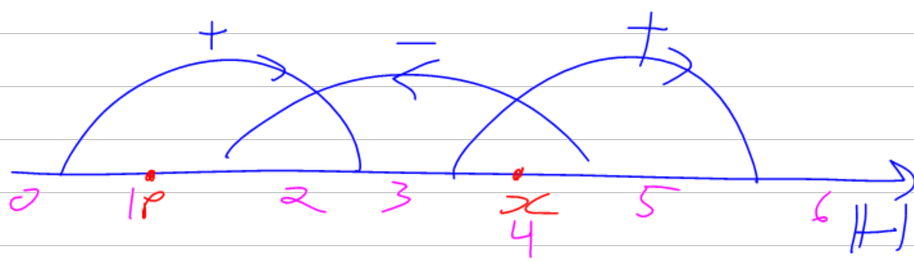
	π_j	ξ_j	p_j	x_j
π_i	0	$-U$	0	0
ξ_i	$-U$	0	0	0
p_i	0	0	0	$R^{\pm}_{ij} \to (1-J^{\pm})^{(p_i-p_j)x_j}$
x_j	0	0		0

$$U = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left(F(1-GF)^{-1}\right)^{T} = (1-FG)^{-1}F$$

$$e^{(J^{-1}-1)(p_i-p_j)x_j}$$



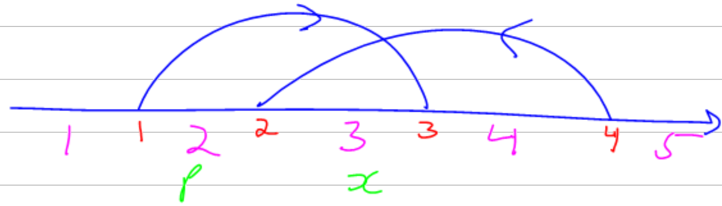


$$\langle p_1 x_4 \rangle = \frac{\text{expression above}}{\text{pure exp.}}$$

$$\langle p_i x_j \rangle = C_{ij}$$

$$A \cdot C = I \qquad A \cdot C = 0$$

$$\left[e^{p\otimes x}, x\otimes 1\right] = e^{p\otimes x}(1\otimes x)$$



$$R^s_{ij} = T^{s/2} e^{(T^s-1)(p_i-p_j)x_j}$$

$$xp = px - 1 \Rightarrow$$

$$G_{\alpha\beta} = \langle p_\alpha x_\beta \rangle \qquad G_{\alpha,2n+1} = 0 \quad \text{\& with effect:} \quad G_{1,\beta} = 0$$

$$\tilde{G}_{\alpha\beta} = \langle x_\beta p_\alpha \rangle = G_{\alpha\beta} - \delta_{\alpha\beta}$$

X^s_{ij}:

make $B \in M_{2n\times(2n+1)}$

$$\begin{cases} \text{row } i & \tilde{G}_{i,\beta} - G_{i+1,\beta} = 0 \iff G_{i\beta} - G_{i+1,\beta} = \delta_{i\beta} \\ \text{row } j & \tilde{G}_{j\beta} - G_{j+1,\beta} - (T^s-1)(G_{i+1,\beta} - \tilde{G}_{i\beta}) = 0 \end{cases}$$

$$\iff T^s G_{j\beta} - G_{j+1,\beta} + (1-T^s) G_{i+1,\beta} = T^s \delta_{j\beta}$$

$$B = (\phi \mid A) \qquad G = \begin{pmatrix} 0 \; 0 \; 0 & 0 \\ D & \vdots \\ & 0 \end{pmatrix} \qquad BG = (I_{2n\times 2n} \mid 0) \qquad AD = I$$

```
PAB1[K_] := Module[{rvk, n, B, c, s, i, j, ρ0, G, ρ1},
  rvk = RVK[K]; n = Length[rvk[[1]]]; B = Table[0, 2 n, 2 n + 1];
  Do[
   s = If[Head[c] === Xp, 1, -1]; {i, j} = List @@ c;
   B[[i, {i, i + 1}]] = {1, -1}; B[[j, {j, j + 1, i + 1}]] = {1, -T^-s, T^-s - 1},
   {c, rvk[[1]]}];
  ρ0 = Det[B[[;; , 2 ;;]]] // Factor;
  G = Table[0, 2 n, 2 n]; G[[;; , ;; 2 n - 1]] = Factor@Inverse[B[[;; , 2 ;;]]][[;; , 2 ;; 2 n]];
  ρ1 = Factor@Plus[
     Sum[
      s = If[Head[c] === Xp, 1, -1]; {i, j} = List @@ c;
      s ((1 - T^s)/2 G[[i, i]] G[[i, j]] - 1/2 (G[[i, i]] G[[j, j]] + G[[i, j]] G[[j, i]]) + (T^(2 s) - T^s)/2 G[[i, j]]^2 + T^s (G[[i, j]] G[[j, j]])),
      {c, rvk[[1]]}],
     Sum[rvk[[2, k]]/2 G[[k - 1, k - 1]], {k, 2, 2 n}]];
  {ρ0, ρ1}]
```