

```
In[*]:= Δ[p_, q_] := Factor[ $\frac{(t^{pq} - 1)(t - 1)}{(t^p - 1)(t^q - 1)}$ ]
```

```
In[*]:= Δ[7, 3]
```

```
Out[*]= 1 - t + t3 - t4 + t6 - t8 + t9 - t11 + t12
```

```
In[*]:= u = Table[0, n, n];
Feed[v_] := Module[{i, vp},
  i = 1; While[i ≤ n ∧ v[[i]] == 0, ++i];
  If[i ≤ n,
    vp =  $\frac{v}{v[[i]}}$ ;
    If[u[[i]] === Table[0, n], u[[i]] = vp, Feed[vp - u[[i]]]]
  ]
```

```
In[*]:= Feed[{0, 1, 1}]; u
```

```
Out[*]= {{0, 0, 0}, {0, 1, 1}, {0, 0, 0}}
```

```
In[*]:= Feed[{1, 0, 1}]; u
```

```
Out[*]= {{1, 0, 1}, {0, 1, 1}, {0, 0, 0}}
```

```
In[*]:= Feed[{1, 1, 0}]; u
```

```
Out[*]= {{1, 0, 1}, {0, 1, 1}, {0, 0, 1}}
```

```
In[*]:= n = 10;
mat = Table[(j + i2 - i)3, {i, n}, {j, n}];
mat // MatrixForm
```

```
Out[*]//MatrixForm=
```

1	8	27	64	125	216	343	512	729	1000
27	64	125	216	343	512	729	1000	1331	1728
343	512	729	1000	1331	1728	2197	2744	3375	4096
2197	2744	3375	4096	4913	5832	6859	8000	9261	10648
9261	10648	12167	13824	15625	17576	19683	21952	24389	27000
29791	32768	35937	39304	42875	46656	50653	54872	59319	64000
79507	85184	91125	97336	103823	110592	117649	125000	132651	140608
185193	195112	205379	216000	226981	238328	250047	262144	274625	287496
389017	405224	421875	438976	456533	474552	493039	512000	531441	551368
753571	778688	804357	830584	857375	884736	912673	941192	970299	1000000

```
In[*]:= u = Table[0, n, n];
Feed[v_] := Module[{i, vp},
  i = 1; While[i ≤ n ∧ v[[i]] == 0, ++i];
  If[i ≤ n,
    vp = v / v[[i]];
    If[u[[i]] == Table[0, n], u[[i]] = vp, Feed[vp - u[[i]]]]
  ]
```

```
In[*]:= Feed[mat[[1]]; MatrixForm[u]
```

Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & 8 & 27 & 64 & 125 & 216 & 343 & 512 & 729 & 1000 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[*]:= Feed[mat[[2]]; MatrixForm[u]
```

Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & 8 & 27 & 64 & 125 & 216 & 343 & 512 & 729 & 1000 \\ 0 & 1 & \frac{151}{38} & \frac{189}{19} & \frac{379}{19} & 35 & \frac{2133}{38} & \frac{1603}{19} & \frac{2294}{19} & \frac{3159}{19} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[*]:= Feed[mat[[3]]; MatrixForm[u]
```

Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & 8 & 27 & 64 & 125 & 216 & 343 & 512 & 729 & 1000 \\ 0 & 1 & \frac{151}{38} & \frac{189}{19} & \frac{379}{19} & 35 & \frac{2133}{38} & \frac{1603}{19} & \frac{2294}{19} & \frac{3159}{19} \\ 0 & 0 & 1 & \frac{330}{89} & \frac{786}{89} & \frac{1520}{89} & \frac{2595}{89} & \frac{4074}{89} & \frac{6020}{89} & \frac{8496}{89} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[*]:= Feed[mat[[4]]; MatrixForm[u]

Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & 8 & 27 & 64 & 125 & 216 & 343 & 512 & 729 & 1000 \\ 0 & 1 & \frac{151}{38} & \frac{189}{19} & \frac{379}{19} & 35 & \frac{2133}{38} & \frac{1603}{19} & \frac{2294}{19} & \frac{3159}{19} \\ 0 & 0 & 1 & \frac{330}{89} & \frac{786}{89} & \frac{1520}{89} & \frac{2595}{89} & \frac{4074}{89} & \frac{6020}{89} & \frac{8496}{89} \\ 0 & 0 & 0 & 1 & 4 & 10 & 20 & 35 & 56 & 84 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[*]:= Feed[mat[[5]]; MatrixForm[u]

Out[*]//MatrixForm=

$$\begin{pmatrix} 1 & 8 & 27 & 64 & 125 & 216 & 343 & 512 & 729 & 1000 \\ 0 & 1 & \frac{151}{38} & \frac{189}{19} & \frac{379}{19} & 35 & \frac{2133}{38} & \frac{1603}{19} & \frac{2294}{19} & \frac{3159}{19} \\ 0 & 0 & 1 & \frac{330}{89} & \frac{786}{89} & \frac{1520}{89} & \frac{2595}{89} & \frac{4074}{89} & \frac{6020}{89} & \frac{8496}{89} \\ 0 & 0 & 0 & 1 & 4 & 10 & 20 & 35 & 56 & 84 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[*]:= NoFeed[v_] := Catch[Module[{i, vp},
  i = 1; While[i ≤ n ∧ v[[i]] == 0, ++i];
  If[i ≤ n,
    vp =  $\frac{v}{v[[i]]}$ ;
    If[u[[i]] === Table[0, n], Throw[NotInSpan], NoFeed[vp - u[[i]]],
      Throw[InSpan]]
  ]]
```

In[*]:= NoFeed[Table[(j + 1)⁴ - j⁴, {j, n}]]

Out[*]=
InSpan

In[*]:= NoFeed[Table[(j + 1)⁵ - j⁵, {j, n}]]

Out[*]=
NotInSpan