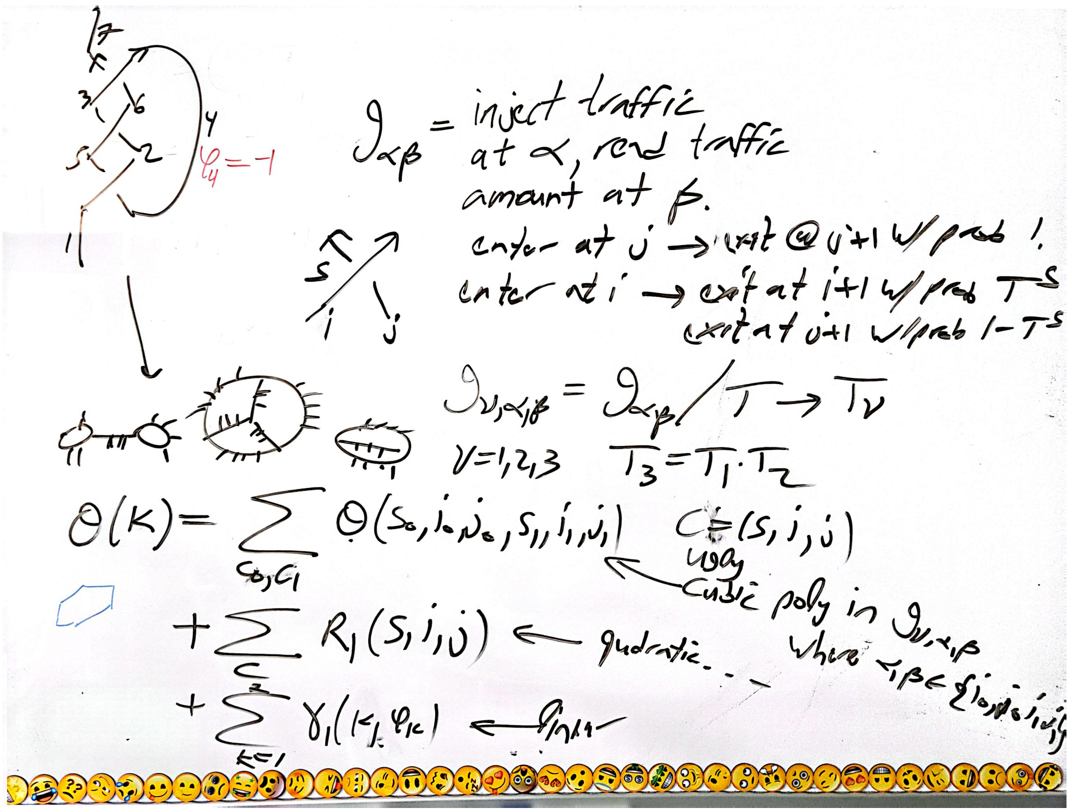
In[∘]:=

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\Talks\\KnotAtLunch\\240830"];
<< KnotTheory`
```

```
C:\drorbn\AcademicPensieve\Projects\KnotTheory\KnotTheory
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

SetDelayed: Tag Diff in Diff[K_PD, rut_, ag_, n_, m_] is Protected.



In[∘]:=
```
Rot[pd_PD] := Module[{n, xs, x, rots, Xp, Xm, front = {1}, k},
    n = Length@pd; rots = Table[0, {2 n}];
    xs = Cases[pd, x_X :> { Xp[x⟦4⟧, x⟦1⟧]  PositiveQ@x
                            Xm[x⟦2⟧, x⟦1⟧]  True ];
    For[k = 1, k ≤ 2 n, ++k,
     If[FreeQ[front, -k],
      front = Flatten@Replace[front, k → (xs /. {
              Xp[k, l_] | Xm[l_, k] :> {l + 1, k + 1, -l},
              Xp[l_, k] | Xm[k, l_] :> (++rots⟦l⟧; {-l, k + 1, l + 1}),
              _Xp | _Xm :> {}
             }), {1}],
      Cases[front, k | -k] /. {k, -k} :> --rots⟦k⟧;
     ]
    ];
    {xs /. {Xp[i_, j_] :> {+1, i, j}, Xm[i_, j_] :> {-1, i, j}}, rots} ];
Rot[K_] := Rot[PD[K]];
```

In[∘]:=
```
Rot[Knot[3, 1]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

Out[∘]=
```
{{{-1, 4, 1}, {-1, 6, 3}, {-1, 2, 5}}, {0, 0, 0, -1, 0, 0}}
```

In[∘]:=
```
CF[ℰ_] := Module[{vs = Union@Cases[ℰ, g__, ∞], ps, c},
    Total[CoefficientRules[Expand[ℰ], vs] /. (ps_ → c_) :> Factor[c] (Times @@ vs^ps)] ];
```

In[∘]:=
```
T3 = T1 T2;
```

In[∘]:=

$$R_1[s\_,i\_,j\_]=\mathrm{CF}\Big[s\,\Big(1/2-g_{3ii}+T_2^s\,g_{1ii}\,g_{2ji}-g_{1ii}\,g_{2jj}-\left(T_2^s-1\right)g_{2ji}\,g_{3ii}+2\,g_{2jj}\,g_{3ii}-\left(1-T_3^s\right)g_{2ji}\,g_{3ji}-g_{2ii}\,g_{3jj}-T_2^s\,g_{2ji}\,g_{3jj}+g_{1ii}\,g_{3jj}+\Big(\left(T_1^s-1\right)g_{1ji}\left(T_2^{2s}\,g_{2ji}-T_2^s\,g_{2jj}+T_2^s\,g_{3jj}\right)+\left(T_3^s-1\right)g_{3ji}\left(1-T_2^s\,g_{1ii}-\left(T_1^s-1\right)\left(T_2^s+1\right)g_{1ji}+\left(T_2^s-2\right)g_{2jj}+g_{2ij}\right)\Big)\Big/\left(T_2^s-1\right)\Big)\Big]$$

Out[∘]=

$$\frac{s}{2}+s\,T_2^s\,g_{1,i,i}\,g_{2,j,i}+\frac{s\left(-1+T_1^s\right)T_2^{2s}\,g_{1,j,i}\,g_{2,j,i}}{-1+T_2^s}-s\,g_{1,i,i}\,g_{2,j,j}-\frac{s\left(-1+T_1^s\right)T_2^s\,g_{1,j,i}\,g_{2,j,j}}{-1+T_2^s}-s\,g_{3,i,i}-$$
$$s\left(-1+T_2^s\right)g_{2,j,i}\,g_{3,i,i}+2\,s\,g_{2,j,j}\,g_{3,i,i}+\frac{s\left(-1+\left(T_1\,T_2\right)^s\right)g_{3,j,i}}{-1+T_2^s}-\frac{s\,T_2^s\left(-1+\left(T_1\,T_2\right)^s\right)g_{1,i,i}\,g_{3,j,i}}{-1+T_2^s}-$$
$$\frac{s\left(-1+T_1^s\right)\left(1+T_2^s\right)\left(-1+\left(T_1\,T_2\right)^s\right)g_{1,j,i}\,g_{3,j,i}}{-1+T_2^s}+\frac{s\left(-1+\left(T_1\,T_2\right)^s\right)g_{2,i,j}\,g_{3,j,i}}{-1+T_2^s}+$$
$$s\left(-1+\left(T_1\,T_2\right)^s\right)g_{2,j,i}\,g_{3,j,i}+\frac{s\left(-2+T_2^s\right)\left(-1+\left(T_1\,T_2\right)^s\right)g_{2,j,j}\,g_{3,j,i}}{-1+T_2^s}+$$
$$s\,g_{1,i,i}\,g_{3,j,j}+\frac{s\left(-1+T_1^s\right)T_2^s\,g_{1,j,i}\,g_{3,j,j}}{-1+T_2^s}-s\,g_{2,i,i}\,g_{3,j,j}-s\,T_2^s\,g_{2,j,i}\,g_{3,j,j}$$

In[∘]:=

$$\theta[\{s0\_,i0\_,j0\_\},\{s1\_,i1\_,j1\_\}]:=\mathrm{CF}\Big[s1\left(T_1^{s0}-1\right)\left(T_2^{s1}-1\right)^{-1}\left(T_3^{s1}-1\right)g_{1,j1,i0}\,g_{3,j0,i1}\left(\left(T_2^{s0}\,g_{2,i1,i0}-g_{2,i1,j0}\right)-\left(T_2^{s0}\,g_{2,j1,i0}-g_{2,j1,j0}\right)\right)\Big]$$

In[∘]:=

$$\Gamma_1[\varphi\_,k\_]=-\varphi/2+\varphi\,g_{3kk}$$

Out[∘]=

$$-\frac{\varphi}{2}+\varphi\,g_{3,k,k}$$

In[∘]:=

```
θ[K_] := Module[{Cs, φ, n, A, s, i, j, k, Δ, G, ν, α, β, gEval, c, z},
  {Cs, φ} = Rot[K]; n = Length[Cs];
  A = IdentityMatrix[2 n + 1];
  Cases[Cs, {s_, i_, j_} :> (A〚{i, j}, {i + 1, j + 1}〛 += ( -T^s  T^s - 1 ; 0  -1 ))];
  Δ = T^((-Total[φ] - Total[Cs〚All,1〛])/2) Det[A];
  G = Inverse[A];
  gEval[ℰ_] := Factor[ℰ /. g_{ν_,α_,β_} :> (G〚α, β〛 /. T → T_ν)];
  z = gEval[Σ_{k1=1}^n Σ_{k2=1}^n θ[Cs〚k1〛, Cs〚k2〛]];
  z += gEval[Σ_{k=1}^n R_1 @@ Cs〚k〛];
  z += gEval[Σ_{k=1}^{2 n} Γ_1[φ〚k〛, k]];
  {Δ, (Δ /. T → T_1) (Δ /. T → T_2) (Δ /. T → T_3) z} // Factor ];
```

In[∘]:=
```
Expand[θ[Knot[3, 1]]]
```

Out[∘]=

$$\left\{-1+\frac{1}{T}+T,\ -\frac{1}{T_1^2}-T_1^2-\frac{1}{T_2^2}-\frac{1}{T_1^2 T_2^2}+\frac{1}{T_1 T_2^2}+\frac{1}{T_1^2 T_2}+\frac{T_1}{T_2}+\frac{T_2}{T_1}+T_1^2 T_2-T_2^2+T_1 T_2^2-T_1^2 T_2^2\right\}$$

In[∘]:=
```
Expand[θ[Knot[10, 165]]]
```

Out[∘]=

$$\Big\{-15-\frac{2}{T^2}+\frac{10}{T}+10\,T-2\,T^2,$$

$$-1404-\frac{9}{T_1^4}-\frac{178}{T_1^3}+\frac{607}{T_1^2}+\frac{624}{T_1}+624\,T_1+607\,T_1^2-178\,T_1^3-9\,T_1^4-\frac{9}{T_2^4}-\frac{9}{T_1^4 T_2^4}+\frac{44}{T_1^3 T_2^4}-$$

$$\frac{65}{T_1^2 T_2^4}+\frac{44}{T_1 T_2^4}-\frac{178}{T_2^3}+\frac{44}{T_1^4 T_2^3}-\frac{178}{T_1^3 T_2^3}+\frac{104}{T_1^2 T_2^3}+\frac{104}{T_1 T_2^3}+\frac{44\,T_1}{T_2^3}+\frac{607}{T_2^2}-\frac{65}{T_1^4 T_2^2}+\frac{104}{T_1^3 T_2^2}+\frac{607}{T_1^2 T_2^2}-$$

$$\frac{1041}{T_1 T_2^2}+\frac{104\,T_1}{T_2^2}-\frac{65\,T_1^2}{T_2^2}+\frac{624}{T_2}+\frac{44}{T_1^4 T_2}+\frac{104}{T_1^3 T_2}-\frac{1041}{T_1^2 T_2}+\frac{624}{T_1 T_2}-\frac{1041\,T_1}{T_2}+\frac{104\,T_1^2}{T_2}+\frac{44\,T_1^3}{T_2}+$$

$$624\,T_2+\frac{44\,T_2}{T_1^3}+\frac{104\,T_2}{T_1^2}-\frac{1041\,T_2}{T_1}+624\,T_1 T_2-1041\,T_1^2 T_2+104\,T_1^3 T_2+44\,T_1^4 T_2+$$

$$607\,T_2^2-\frac{65\,T_2^2}{T_1^2}+\frac{104\,T_2^2}{T_1}-1041\,T_1 T_2^2+607\,T_1^2 T_2^2+104\,T_1^3 T_2^2-65\,T_1^4 T_2^2-178\,T_2^3+\frac{44\,T_2^3}{T_1}+$$

$$104\,T_1 T_2^3+104\,T_1^2 T_2^3-178\,T_1^3 T_2^3+44\,T_1^4 T_2^3-9\,T_2^4+44\,T_1 T_2^4-65\,T_1^2 T_2^4+44\,T_1^3 T_2^4-9\,T_1^4 T_2^4\Big\}$$

In[∘]:=
```
PolyPlot[0] = Graphics[{}];
PolyPlot[p_] := Module[{crs, m1, m2, maxc, minc, s, hex},
   crs = CoefficientRules[T1^(m1=-Exponent[p,T1,Min]) T2^(m2=-Exponent[p,T2,Min]) p, {T1, T2}];
   maxc = N@Log@Max@Abs[Last /@ crs];
   minc = N@Log@Min@Select[Abs[Last /@ crs], # > 0 &];
   If[minc == maxc, s[_] = 0, s[c_] := s[c] = (maxc - Log@c) / (maxc - minc)];
   hex = Table[{Cos[α], Sin[α]} / Cos[2 π / 12] / 2, {α, 2 π / 12, 2 π, 2 π / 6}];
   Graphics[crs /. ({x1_, x2_} → c_) :→ {
       If[c == 0, White, Lighter[Which[
           c > 0 ∧ OddQ[c], Red,
           c > 0 ∧ EvenQ[c], Green,
           c < 0 ∧ OddQ[c], Yellow,
           c < 0 ∧ EvenQ[c], Blue
          ], 0.88 s[Abs@c]]],
       Polygon[(({{1, -1/2}, {0, Sqrt[3]/2}}).{x1 + m1, x2 + m2} + #) & /@ hex] }] ];
PolyPlot[{Δ_, θ_}] := PolyPlot[θ]
```

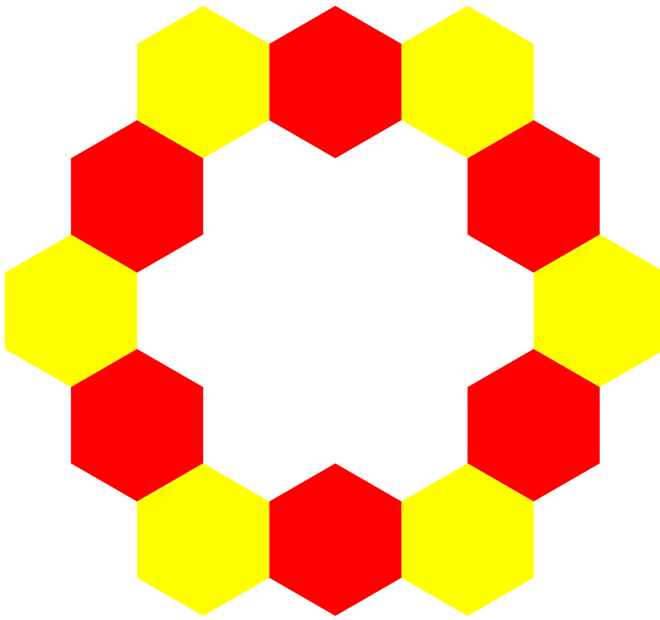
In[ ]:= **θ[Knot[3, 1]][[2]] // Expand**

Out[ ]=

$$-\frac{1}{T_1^2} - T_1^2 - \frac{1}{T_2^2} - \frac{1}{T_1^2 T_2^2} + \frac{1}{T_1 T_2^2} + \frac{1}{T_1^2 T_2} + \frac{T_1}{T_2} + \frac{T_2}{T_1} + T_1^2 T_2 - T_2^2 + T_1 T_2^2 - T_1^2 T_2^2$$

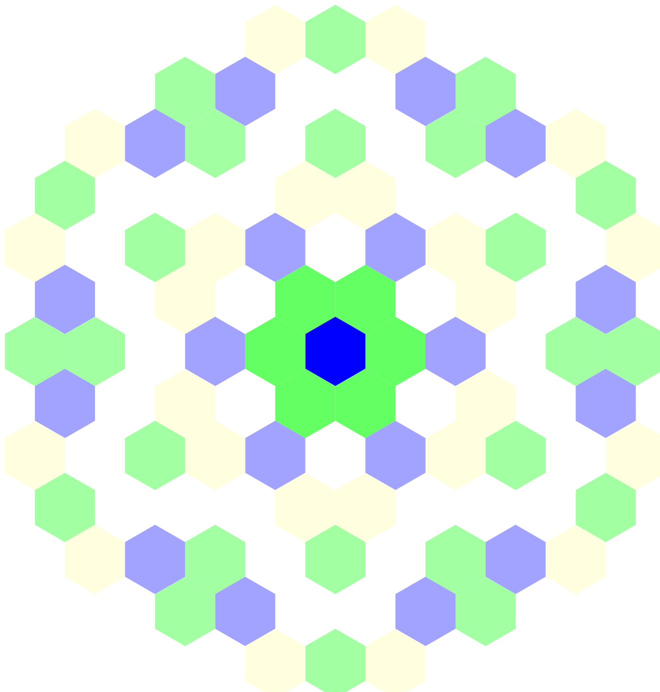
In[ ]:= **PolyPlot[θ[Knot[3, 1]]]**

Out[ ]=



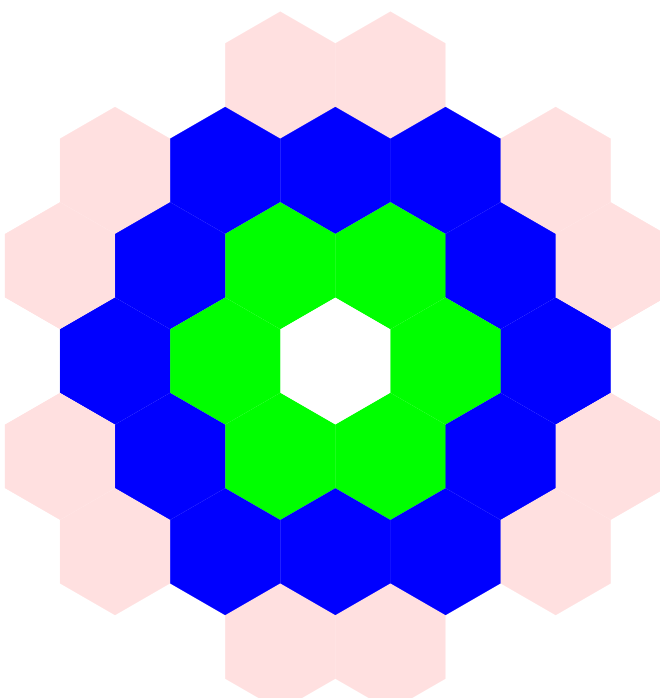
In[∘]:= PolyPlot[θ[Knot["K11n34"]]]

Out[∘]=



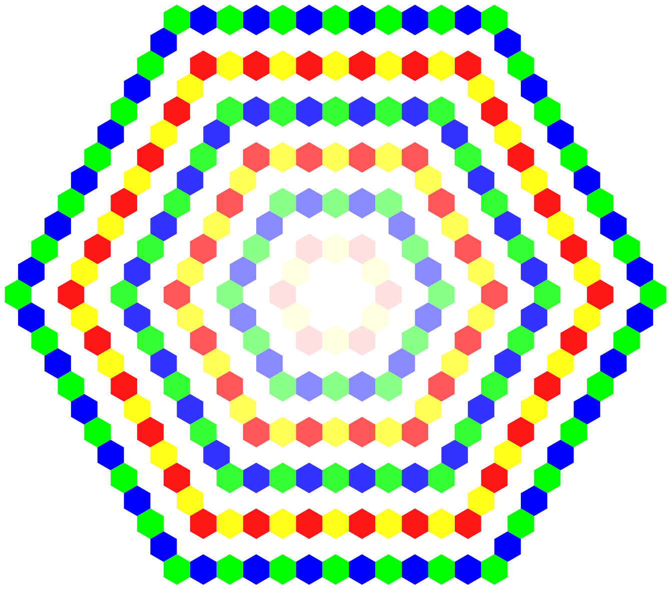
In[∘]:= PolyPlot[θ[Knot["K11n42"]]]

Out[∘]=



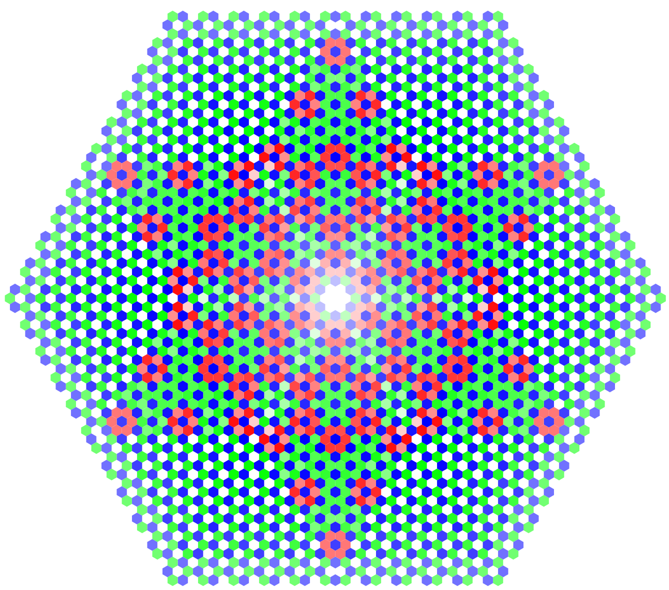
In[•]:= PolyPlot[θ[TorusKnot[13, 2]]]

Out[•]=



In[•]:= PolyPlot[θ[TorusKnot[17, 3]]]

Out[•]=

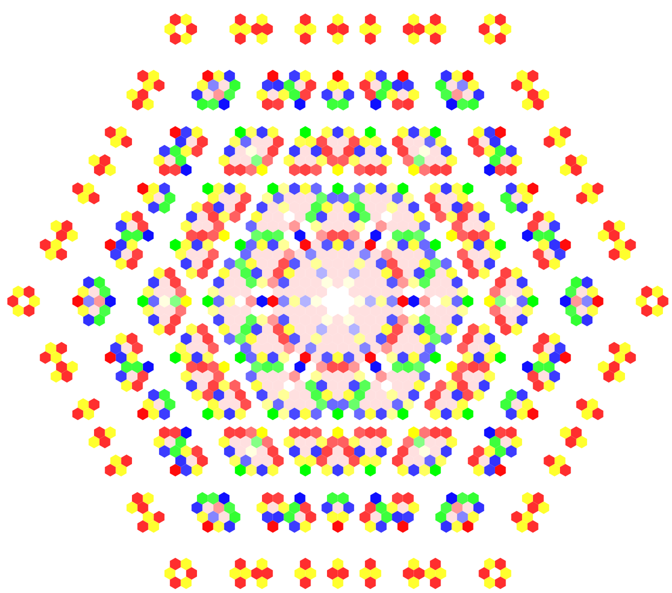


In[∘]:= `PolyPlot[θ[TorusKnot[13, 5]]]`

Out[∘]=

In[∘]:= `PolyPlot[θ[TorusKnot[7, 6]]]`

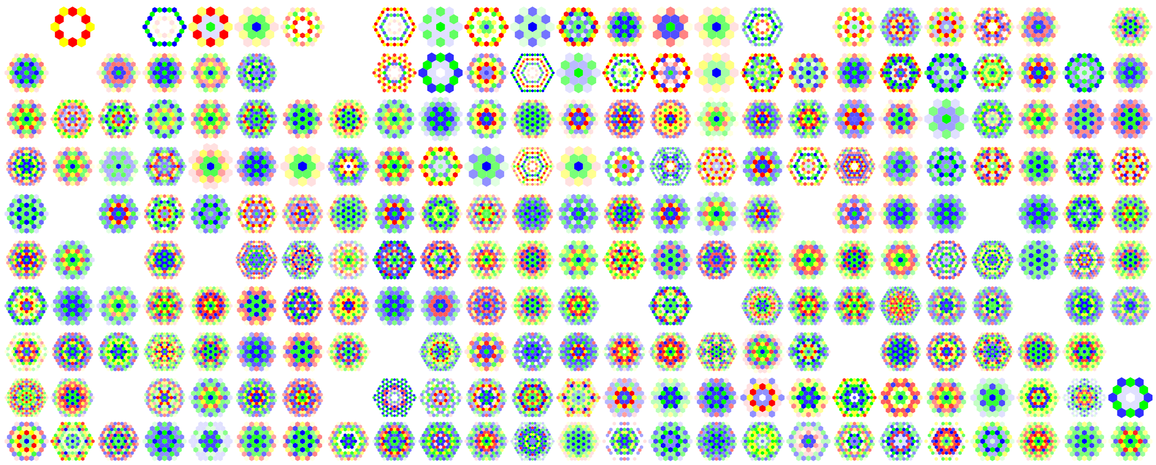
Out[∘]=



In[∘]:= `tab250 = {0}~Join~Table[θ[K]⟦2⟧, {K, AllKnots[{3, 10}]}];`

In[]:= **g250 = GraphicsGrid[Partition[PolyPlot /@ tab250, 25], Spacings → 0]**

Out[]=



In[]:= **Export["g250.png", g250]**

Out[]=

g250.png

See also https://drorbn.net/AcademicPensieve/Projects/HigherRank/DunfieldKnots/.

In[]:= **θ[BR[5, {1, 2, 3, -1, 2, 1, 3}]]**

Out[]=

$$\left\{\frac{2-3T+2T^2}{T},\ \frac{1}{T_1^2 T_2^2}\left(9-13T_1+9T_1^2-13T_2+6T_1T_2+6T_1^2T_2-13T_1^3T_2+9T_2^2+6T_1T_2^2-12T_1^2T_2^2+6T_1^3T_2^2+9T_1^4T_2^2-13T_1T_2^3+6T_1^2T_2^3+6T_1^3T_2^3-13T_1^4T_2^3+9T_1^2T_2^4-13T_1^3T_2^4+9T_1^4T_2^4\right)\right\}$$

θ[BR[5, {1, 2, 3, -1, 2, 1, 3}]]