This is the twisted Drinfeld double (co)acting on one half of itself. As a Yetter Drinfeld object.

Next we set out to generalize to a quantum Burau, following [LNvdV, GK].

The Burau matrix for a generator is $\begin{pmatrix} 1-t & 1 \\ t & 0 \end{pmatrix}$ padded with diagonal identity blocks.

Concrete example: braided line

$B = \mathbb{Q}[x] \quad S[x] = qx, \Delta(x) = 1 \otimes x + x \otimes 1$

$\Psi(x \otimes x) = qx \otimes x \quad \varphi(x) = tx \quad \eta = 1, \varepsilon(x) = 0$

References:

[BNvdV] Bar-Natan, van der Veen, *Perturbed Gaussian generating functions for universal knot invariants,* Arxiv 2109.02057.

[GK] Kashaev, Garoufalidis, *Multivariable knot polynomials from braided Hopf algebras with automorphisms* Arxiv 2311.11528.

[LNvdV] Lopez-Neumann, van der Veen, *Genus bounds for twisted quantum invariants,* JEMS 2023, Arxiv 2211.15010.

[MK] Murakami, Korinman, *Relating quantum character varieties and skein modules,* Arxiv 2211.04252.

[MV] Murakami, van der Veen, *Quantized representations of knot groups*, Quantum topology (2023). Arxiv 1812.09539

Using exponential generating functions as in [BNvdV] we can explicitly compute everything in this example as a series in $\hbar$ where $q = e^{\hbar}$. This should yield the full loop expansion. In particular inverse Alexander when q = 1 and rho_1 when q = 1+h.

The value of the R-matrix (as a map $B^{\otimes 2} \to B^{\otimes 2}$) is (with dual variables ξ_i)

$$e^{(1-t)\xi_i x_j + \xi_i x_i + t\xi_j x_j} \Big(1 + \left(\tfrac{1}{2}\left(t^2 - t\right) x_j^2 \xi_i^2 + \tfrac{1}{2}(1-3t) x_i x_j \xi_i^2 + t x_i x_j \xi_j \xi_i\right)\hbar + O\left(\hbar^2\right)\Big)$$

That's Burau!

Working with half de double allows for easier generalization, possibly more examples in higher rank. Presumably all one needs is a braided Hopf algebra with an automorphism. Gaussians techniques persist. Hopefully the present form of rho_1 is sufficiently simple to infer some topological properties beyond genus such as, fiberedness, ribbonness etc.