Burau, Bottom tangles, Braided Hopf algebra
Roland van der Veen, Toronto 12-2-2024

The braid group acts as diffeomorphisms of the punctured disk D. By passing from homotopy classes of arcs to isotopy classes in a thickening of D leads us from the Artin and Burau representations to more recent constructions of quantum invariants including rho1 by [BNvdV].
An essential role is played by the cyclic cover of D


Burau Artin Habiro Bar-Natan Weenink Garoufalidis Kashaev

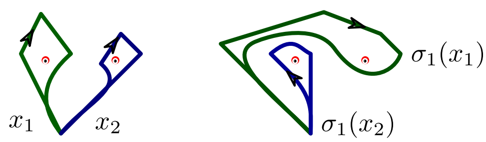
for more on Burau see the next sheet!

Burau representation:

The total winding number around the punctures gives $\alpha : \pi_1(D,p) \to \langle t \rangle$ and the kernel yields the cyclic cover $\tilde{D}$ of D. Braids now act on $H_1(\tilde{D},\tilde{p};\mathbb{Z})$ viewed as a $\mathbb{Z}[t,t^{-1}]$ module where t acts by going up one sheet. (Apply t^{-1} to the handout to read more....

Artin representation:

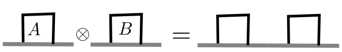
The braid generators σ_i
act on the fundamental group generators x_i
by $\sigma_i(x_i) = x_i x_{i+1} x_i^{-1}$ $\sigma_i(x_{i+1}) = x_i$
and trivially otherwise.



Category $\mathcal{T}$ of bottom tangles handlebodies

Habiro: 'Quantum fundamental group' [MK,MV]

$\mathrm{Obj}(\mathcal{T}) = \mathbb{N}$ $\quad \mathcal{T}_{h,s} =$ Linear combinations of s-strand tangles in ball with h holes with adjacent ends on the bottom line



Composition: A∘B is the result of treating the handles of B as 'cable management sleeves' and identifying each sleeve of B with the corresponding strand of A.

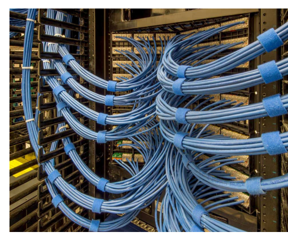


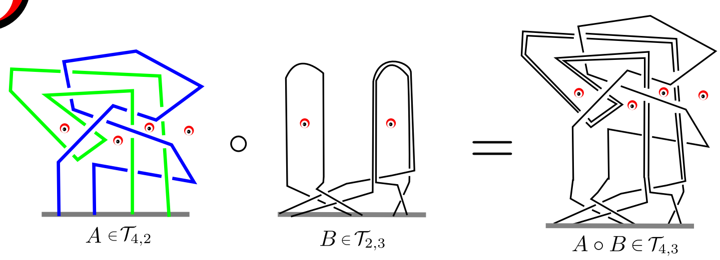
Image courtesy of cable management, Speedtech international.

More braided Hopf

And the braid group action can be written Hopf algebraically!

$$\delta^i_{j,k} = m_j^{1,3} \circ \Psi_{k,3} \circ \phi_3 \circ S_3 \circ \Delta^i_{1,k,3}$$

$$R_{i,j} = m_j^{1,j} \circ \Psi_{i,j} \circ \delta^i_{1,i} \circ \varphi_j$$



1 is a braided Hopf object in $\mathcal{T}$

For example: $m \circ (\mathrm{id} \otimes S) \circ \Delta = \varepsilon\eta$

$$\Delta \circ m = (m \otimes m) \circ (\mathrm{id} \otimes \Psi \otimes \mathrm{id}) \circ (\Delta \otimes \Delta)$$