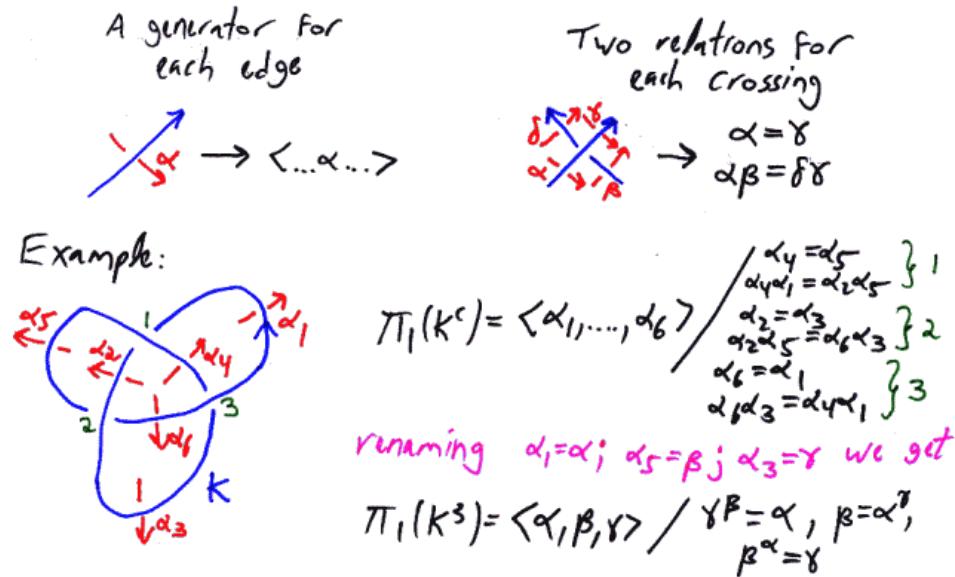




Problem 1 (30 points). Let K be a knot in \mathbb{R}^3 presented by a planar diagram D . With a massive use of van Kampen's theorem, show that the fundamental group of the complement of K has a presentation (the “Wirtinger presentation”, as discussed in class) with one generator for each edge of D and two relations for each crossing of D , as indicated in the figure below.



(There will be a brief discussion of this question in class on Monday Feb 9).

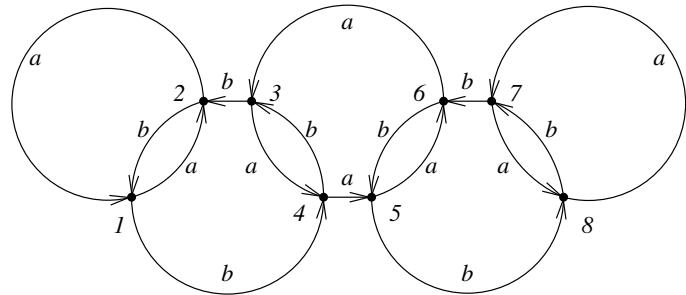
Problem 2 (15 points). The trefoil knot above, whose fundamental group is $G_1 = \langle \alpha, \beta, \gamma: \alpha = \gamma^\beta, \beta = \alpha^\gamma, \gamma = \beta^\alpha \rangle$ is in fact the torus knot $T_{3/2}$, whose fundamental group, as computed in class, is $G_2 = \langle \lambda, \mu: \lambda^2 = \mu^3 \rangle$. Prevent the collapse of mathematics by showing that these two groups are isomorphic.

Problem 3 (10 points). Let X be the “Olympic Rings” covering of the figure 8 space, 8_b^a , whose basepoint is taken to be at the quadrivalent vertex in its centre and whose fundamental group is the free group on two letter a and b : $G := \pi_1(8_b^a) = F(a, b)$.

A. Describe the right G -set S corresponding to 8_b^a : it is a set with elements, and a and b act on it as the permutations and .

B. Taking the basepoint x_1 of X to be the point marked as “1” on the right, write a set of generators for the image H of $\pi_1(X, x_1)$ within $G := \pi_1(8_b^a)$.

C. Is H a normal subgroup of G ?



Problem 4 (10 points). Prove Corollary 11 from the Covering Spaces handout: If X is a connected covering of a nice space B (meaning, B is connected, locally connected and semi-locally simply connected) and $H := p_*\pi_1(X) < G := \pi_1(B)$, then $\text{Aut}(X) = N_G(H)/H$ where $N_G(H) := \{g \in G: H = g^{-1}Hg\}$ is the normalizer of H in G .

Problem 5 (10 points). Describe the universal covering space U of the space B which is the union of a 2-dimensional sphere and one of its diameter lines. (Don't say “it's the space of spelunkers” – you are expected to give a concrete description of U as some familiar space or as a simple subset of some familiar space).

Problem 6 (10 points). If B is a nice space and U its universal cover, show that U is a covering of every connected covering X of B .