

Pensieve header: A guest class in Lisa Jeffrey's UTSC reading course.

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In[*]:= BarcodeImage["https://drorbn.net/AcademicPensieve/Classes/25-GuestAtUTSC"]
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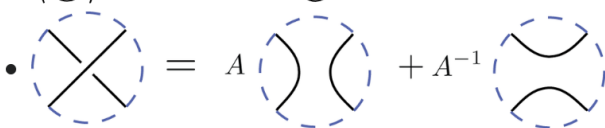
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Out[*]=
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From https://en.wikipedia.org/wiki/Bracket_polynomial:

Definition [\[edit\]](#)

The bracket polynomial of any (unoriented) link diagram L , denoted $\langle L \rangle$, is a polynomial in the variable A , characterized by the three rules:

- $\langle \bigcirc \rangle = 1$, where $\bigcirc$ is the standard diagram of the [unknot](#)
- 
- $\langle \bigcirc \sqcup L \rangle = (-A^2 - A^{-2})\langle L \rangle$

From <https://www.math.uni-hamburg.de/home/runkel/Material/SS20/L1hand.pdf>:

Example 3.2. Let D be the canonical diagram of the Trefoil knot. We calculate the Kaufmann bracket of the Trefoil knot $\langle D \rangle$. At first resolve all crossings with the first formula.

$$\begin{aligned}
 \langle \text{Trefoil} \rangle &= A \langle \text{Trefoil}_1 \rangle + A^{-1} \langle \text{Trefoil}_2 \rangle \\
 &= A^2 \langle \text{Trefoil}_1 \rangle + \langle \text{Trefoil}_1 \rangle + \langle \text{Trefoil}_2 \rangle + A^{-2} \langle \text{Trefoil}_3 \rangle \\
 &= A^3 \langle \text{Trefoil}_1 \rangle + A \langle \text{Trefoil}_1 \rangle + A \langle \text{Trefoil}_2 \rangle + A^{-1} \langle \text{Trefoil}_3 \rangle \\
 &\quad + A \langle \text{Trefoil}_4 \rangle + A^{-1} \langle \text{Trefoil}_5 \rangle + A^{-1} \langle \text{Trefoil}_6 \rangle + A^{-3} \langle \text{Trefoil}_7 \rangle.
 \end{aligned}$$

Since a diagram without crossings is the disjoint union of loops we obtain the value of the bracket recursively by (II) and (III).

$$\langle D \rangle = (-A^2 - A^{-2})(-A^5 - A^{-3} + A^{-7}).$$

In[] := << KnotTheory`

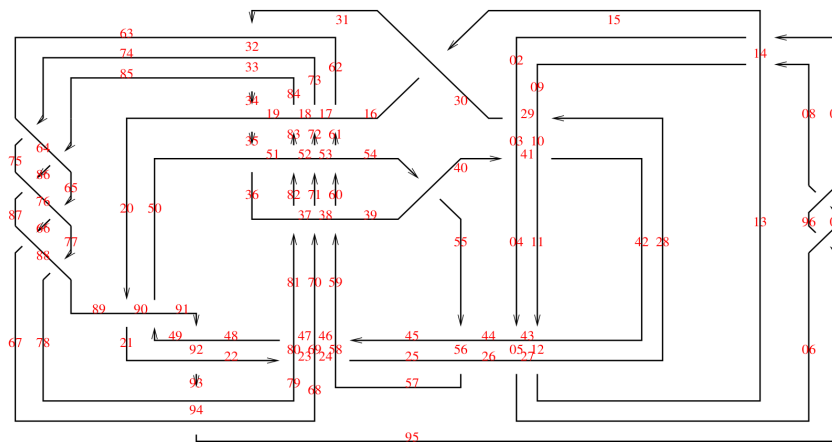
Loading KnotTheory` version of October 29, 2024, 10:29:52.1301.

Read more at <http://katlas.org/wiki/KnotTheory>.

The 48-crossing Gompf-Scharlemann-Thompson knot [GST] is significant because it may be a counterexample to the slice-ribbon conjecture:



Gompf Scharlemann Thompson



[GST] R. E. Gompf, M. Scharlemann, and A. Thompson, *Fibered Knots and Potential Counterexamples to the Property 2R and Slice-Ribbon Conjectures*, *Geom. and Top.* **14** (2010) 2305–2347, [arXiv:1103.1601](https://arxiv.org/abs/1103.1601).

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