# Monsters



**Solving the Linear**, $ax+b=0$: $x=-b/a$:

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In[∘]:= First@Solve[a x + b == 0, x]
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$$\text{Out}[\circ]= \left\{x\to -\frac{b}{a}\right\}$$

**Solving the Quadratic**, $ax^2+bx+c=0$: $\Delta=b^2-4ac$; $\delta=\sqrt{\Delta}$; $x=\frac{\delta-b}{2a}$:

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In[∘]:= First@Solve[a x^2 + b x + c == 0, x]
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$$\text{Out}[\circ]= \left\{x\to \frac{-b-\sqrt{b^2-4ac}}{2a}\right\}$$

**Solving the Cubic**, $ax^3+bx^2+cx+d=0$: $\Delta=27a^2d^2-18abcd+4ac^3+4b^3d-b^2c^2$; $\delta=\sqrt{\Delta}$; $\Gamma=27a^2d-9abc+3\sqrt{3}a\delta+2b^3$; $\gamma=\sqrt[3]{\frac{\Gamma}{2}}$; $x=-\frac{\frac{b^2-3ac}{\gamma}+b+\gamma}{3a}$:

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In[∘]:= First@Solve[a x^3 + b x^2 + c x + d == 0, x]
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$$\text{Out}[\circ]= \left\{x\to -\frac{b}{3a}-\frac{2^{1/3}\left(-b^2+3ac\right)}{3a\left(-2b^3+9abc-27a^2d+\sqrt{4\left(-b^2+3ac\right)^3+\left(-2b^3+9abc-27a^2d\right)^2}\right)^{1/3}}+\frac{\left(-2b^3+9abc-27a^2d+\sqrt{4\left(-b^2+3ac\right)^3+\left(-2b^3+9abc-27a^2d\right)^2}\right)^{1/3}}{3\times 2^{1/3}a}\right\}$$

**Solving the Quartic**, $ax^4+bx^3+cx^2+dx+e=0$: $\Delta_0=12ae-3bd+c^2$; $\Delta_1=-72ace+27ad^2+27b^2e-9bcd+2c^3$; $\Delta_2=\frac{1}{27}\left(\Delta_1^2-4\Delta_0^3\right)$; $u=\frac{8ac-3b^2}{8a^2}$; $v=\frac{8a^2d-4abc+b^3}{8a^3}$; $\delta_2=\sqrt{\Delta_2}$; $Q=\frac{1}{2}\left(3\sqrt{3}\delta_2+\Delta_1\right)$; $q=\sqrt[3]{Q}$; $S=\frac{\frac{\Delta_0}{q}+q}{12a}-\frac{u}{6}$; $s=\sqrt{S}$; $\Gamma=-\frac{v}{s}-4S-2u$; $\gamma=\sqrt{\Gamma}$; $x=-\frac{b}{4a}+\frac{\gamma}{2}+s$:

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In[∘]:= First@Solve[a x^4 + b x^3 + c x^2 + d x + e == 0, x]
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$$\text{Out}[\circ]= \Bigg\{x\to -\frac{b}{4a}-\frac{1}{2}\sqrt{\left(\frac{b^2}{4a^2}-\frac{2c}{3a}+\left(2^{1/3}\left(c^2-3bd+12ae\right)\right)\Big/\left(3a\left(2c^3-9bcd+27ad^2+27b^2e-72ace+\sqrt{-4\left(c^2-3bd+12ae\right)^3+\left(2c^3-9bcd+27ad^2+27b^2e-72ace\right)^2}\right)^{1/3}\right)+\frac{1}{3\times 2^{1/3}a}\left(2c^3-9bcd+27ad^2+27b^2e-72ace+\sqrt{-4\left(c^2-3bd+12ae\right)^3+\left(2c^3-9bcd+27ad^2+27b^2e-72ace\right)^2}\right)^{1/3}\right)}-$$

$$\frac{1}{2}\sqrt{\left(\frac{b^2}{2a^2}-\frac{4c}{3a}-\left(2^{1/3}\left(c^2-3bd+12ae\right)\right)\Big/\left(3a\left(2c^3-9bcd+27ad^2+27b^2e-72ace+\sqrt{-4\left(c^2-3bd+12ae\right)^3+\left(2c^3-9bcd+27ad^2+27b^2e-72ace\right)^2}\right)^{1/3}\right)-\frac{1}{3\times 2^{1/3}a}\left(2c^3-9bcd+27ad^2+27b^2e-72ace+\sqrt{-4\left(c^2-3bd+12ae\right)^3+\left(2c^3-9bcd+27ad^2+27b^2e-72ace\right)^2}\right)^{1/3}-\left(-\frac{b^3}{a^3}+\frac{4bc}{a^2}-\frac{8d}{a}\right)\Big/\left(4\sqrt{\left(\frac{b^2}{4a^2}-\frac{2c}{3a}+\left(2^{1/3}\left(c^2-3bd+12ae\right)\right)\Big/\left(3a\left(2c^3-9bcd+27ad^2+27b^2e-72ace+\sqrt{-4\left(c^2-3bd+12ae\right)^3+\left(2c^3-9bcd+27ad^2+27b^2e-72ace\right)^2}\right)^{1/3}\right)+\frac{1}{3\times 2^{1/3}a}\left(2c^3-9bcd+27ad^2+27b^2e-72ace+\sqrt{-4\left(c^2-3bd+12ae\right)^3+\left(2c^3-9bcd+27ad^2+27b^2e-72ace\right)^2}\right)^{1/3}\right)}\right)\right)}\Bigg\}$$

**Solving the Quintic**, $ax^5+bx^4+cx^3+dx^2+ex+f=0$: An even bigger monster? Galois: Not in this universe.