This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

# Homework Assignment 7



Solve and submit your solutions of the following problems. Note also that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

#### **Due date**

Friday, November 14, 2025 11:59 pm (Eastern Standard Time)

#### Late penalty

5% deducted per hour

#### Q1 (10 points)

Let N and H be groups, and let  $\phi: H \to \operatorname{Aut}(N)$  and  $\psi \in \operatorname{Aut}(H)$  be given, and let  $\phi' := \phi \circ \psi$ . Show that  $N \vartriangleleft_{\phi'} H$  is isomorphic to  $N \vartriangleleft_{\phi} H$ .

### Q2 (10 points)

Let  $N = \langle x \rangle / \langle x^7 \rangle$  and let  $H = \langle y \rangle / \langle y^3 \rangle$ , and let  $\phi_1 : H \to \operatorname{Aut}(N)$  be given by  $\phi_{1y}(x) = x^2$  and  $\phi_2 : H \to \operatorname{Aut}(N)$  be given by  $\phi_{2y}(x) = x^4$ . Show that the groups  $N \lhd_{\phi_i} H$  for i = 1, 2 are isomorphic.

## Q3 (10 points)

Let N, A, and B be groups, and let  $H:=A\times B$ . Suppose  $\psi:A\to \operatorname{Aut}(H)$  is given, and let  $\phi:=\psi\circ\pi_A$  where  $\pi_A:A\times B\to A$  is the projection on the first factor. Show that  $N\vartriangleleft_\phi H$  is isomorphic to  $(N\vartriangleleft_\psi A)\times B$ .

### Q4 (10 points)

Show that up to isomorphism and other then the direct product, there is only one group of the form  $\mathbb{Z}/3 \triangleleft (\mathbb{Z}/2 \times \mathbb{Z}/2)$ .

# Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed.

