This is a preview of what students will see when they are submitting the assignment. Interactive features are disabled.

# Homework Assignment 4

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

**Due date**

Friday, October 10, 2025 11:59 pm (Eastern Daylight Time)

**Late penalty**

5% deducted per hour

## Q1 (10 points)

If $C \lhd A$ and $D \lhd B$, show that $(C \times D) \lhd (A \times B)$ and $(A/C) \times (B/D) \simeq (A \times B)/(C \times D)$.

## Q2 (10 points)

If $A \lhd G$, $B \lhd G$, and $G = AB$, show that $G/(A \cap B) \simeq (G/A) \times (G/B)$.

## Q3 (10 points)

Show that if $A$ is Abelian and simple (though not necessarily finite), then $A \simeq \mathbb{Z}/p$ for some prime $p$.

## Q4 (10 points)

A Group $G$ is called "solvable" if there is a sequence of groups $G_0, G_1, \ldots, G_n$ such that $G = G_0 \rhd G_1 \rhd \ldots \rhd G_n = \{e\}$ and such that $G_i/G_{i+1}$ is Abelian for all $0 \leq i < n$.

1. Show that subgroups and quotient groups of solvable groups are solvable.
2. Show that if $N \lhd G$ and $N$ and $G/N$ are solvable, then so is $G$.

## Q5 (10 points)

Show that if $n \geq 3$ then $A_n$ contains a subgroup isomorphic to $S_{n-2}$.

## Q6 (10 points)

1. Let $G$ be a group such that there is an infinite chain of simple groups $G_1 \leq G_2 \leq G_3 \leq \cdots$ such that $G = \bigcup G_i$. Show that $G$ is simple.
2. Let $A_\infty$ be the set of bijections $\sigma : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ that are equal to the identity beyond some point and that are even up to that point. Namely, for each $\sigma$ there is some $n$ (which may depend on $\sigma$) such that if $k > n$ then $\sigma(k) = k$ and such that $\sigma|_{\underline{n}} \in A_n$, where $A_n$ denotes the $n$th alternating group, a subgroup of $S_n$. Prove that $A_\infty$ is an infinite simple group.

## Ready to submit?

→ Please ensure all pages are in order and rotated correctly before you submit

→ You will not be able to resubmit your work after the due date has passed.

Please wait...