Homework Assignment 2

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date

Friday, September 26, 2025 11:59 pm (Eastern Daylight Time)

Late penalty

5% deducted per hour

Q1 (10 points)

1. Prove that the groups $(\mathbb{Z}, +)$ and $(\mathbb{Q}, +)$ are not isomorphic.
2. Let D_{2n} denote the group of symmetries of a regular n-gon, including rotations and reflections. Prove that the group D_{24} is not isomorphic to S_4.

Q2 (10 points)

Let G be a group. Prove that the map $\iota \colon G \to G$ defined by $\iota(g) = g^{-1}$ is always an anti-isomorphism, yet it is an isomorphism if and only if G is Abelian.

Q3 (10 points)

Describe in as simple terms as you can the automorphism group $\mathrm{Aut}((\mathbb{Q}, +))$.

Q4 (10 points)

If g is an element of a group G, the *order* $|g|$ of g is the least positive number n for which $g^n = 1$ (it may be ∞).

1. Prove that if x and y are conjugate, their orders are the same.
2. Prove that for any x and y, $|xy| = |yx|$.

Q5 (10 points)

Let G be a group. The *commutator* of two elements $a, b \in G$ is defined to be $[a, b] := aba^{-1}b^{-1}$. Let G' be the subgroup of G generated by all the commutators of pairs of elements of G. Show that G' is normal in G, that $A := G/G'$ is Abelian, and that any morphism from G into an Abelian group B factors through A.

$A = G/G'$ is often called "the Abelianization of G".

If you are not sure what "factors through" means, look it up.

Q6 (10 points)

Let G be a group. We've defined a map $C : G \to \mathrm{Aut}(G)$ by $C_h(g) := h^{-1}gh$. The image of this map, $\operatorname{im} C \subset \mathrm{Aut}(G)$, is called "the set of inner automorphisms of G".

Show that the set of inner automorphisms of a group G is a normal subgroup of the set $\mathrm{Aut}(G)$ of all automorphisms of G.

Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed.

Please wait...