

Pensieve Header: Programs to generate pictures of the barycentric subdivision and of the prism construction, written for my summer class on <http://www.math.toronto.edu/~drorbn/classes/0910/Homology> Homology, Accra, Ghana, June 2010.

```
del[s_Simplex] := Table[
  Delete[s, i],
  {i, Length[s]}
];
del[expr_] := expr /. s_Simplex => del[s];
cone[a_, s_Simplex] := Prepend[s, a];
cone[a_, expr_] := expr /. s_Simplex => cone[a, s];
B[Simplex[v_]] := Simplex[v];
B[s_Simplex] := cone[Expand[Mean[List @@ s]], B[del[s]]];
B[expr_] := expr /. s_Simplex => B[s];
```

B[Simplex[e0, e1, e2]]

$$\left\{ \left\{ \text{Simplex} \left[\frac{e0}{3} + \frac{e1}{3} + \frac{e2}{3}, \frac{e1}{2} + \frac{e2}{2}, e2 \right], \text{Simplex} \left[\frac{e0}{3} + \frac{e1}{3} + \frac{e2}{3}, \frac{e1}{2} + \frac{e2}{2}, e1 \right] \right\}, \right.$$

$$\left\{ \text{Simplex} \left[\frac{e0}{3} + \frac{e1}{3} + \frac{e2}{3}, \frac{e0}{2} + \frac{e2}{2}, e2 \right], \text{Simplex} \left[\frac{e0}{3} + \frac{e1}{3} + \frac{e2}{3}, \frac{e0}{2} + \frac{e2}{2}, e0 \right] \right\},$$

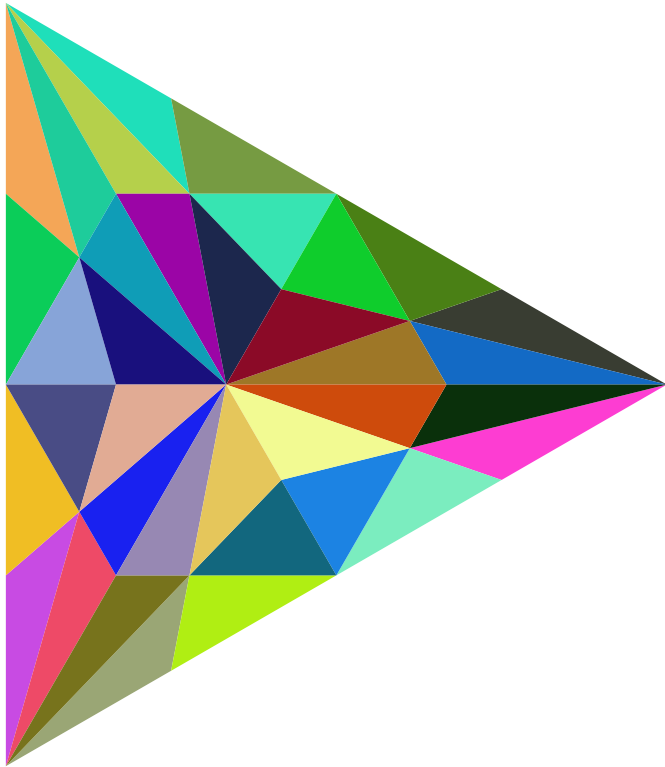
$$\left. \left\{ \text{Simplex} \left[\frac{e0}{3} + \frac{e1}{3} + \frac{e2}{3}, \frac{e0}{2} + \frac{e1}{2}, e1 \right], \text{Simplex} \left[\frac{e0}{3} + \frac{e1}{3} + \frac{e2}{3}, \frac{e0}{2} + \frac{e1}{2}, e0 \right] \right\} \right\}$$

```
Draw2D[ss_] := Module[
  {rule, mat},
  rule = Thread[{e0, e1, e2} ->  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ];
  mat = Drop[Orthogonalize[ $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ], 1];
  Graphics[ss /. s_Simplex => {
    RGBColor[Random[], Random[], Random[]],
    Polygon[mat.(# /. rule) & /@ List @@ s]
  }]
]
```

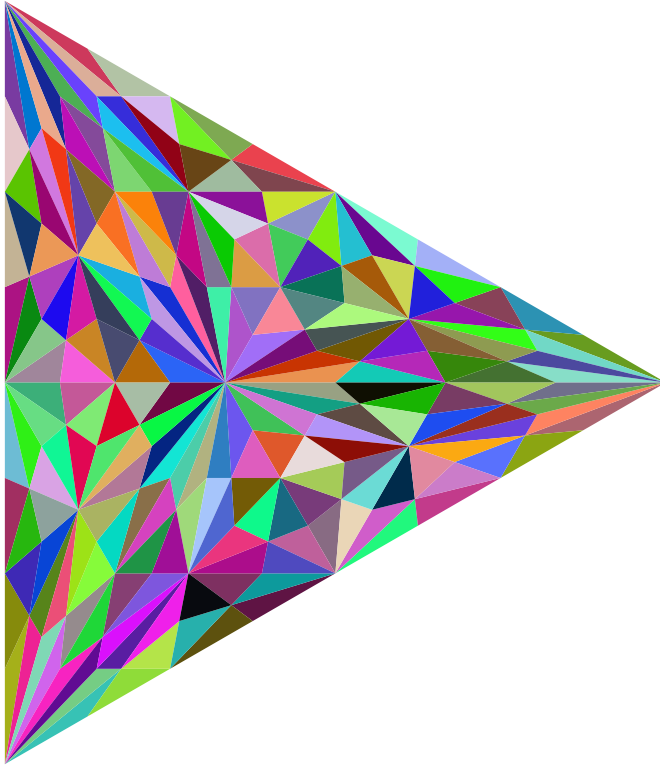
```
b21 = Draw2D[B[Simplex[e0, e1, e2]]]
```



```
b22 = Draw2D[B[B[Simplex[e0, e1, e2]]]]
```

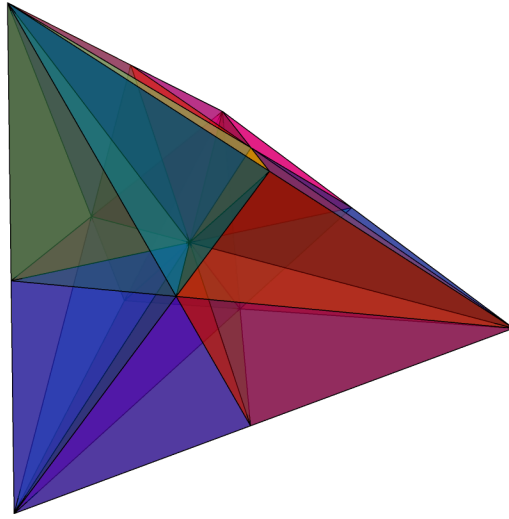


```
b23 = Draw2D[B[B[B[Simplex[e0, e1, e2]]]]]
```

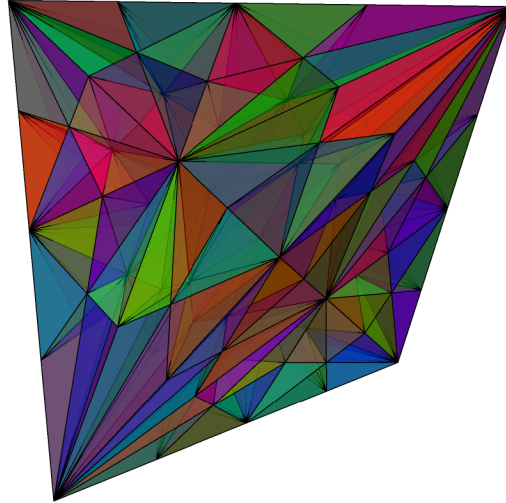


```
Draw3D[ss_] := Module[
  {rule, mat, z, r, t},
  mat = IdentityMatrix[4];
  rule = Thread[{e0, e1, e2, e3} → mat];
  mat = Drop[Orthogonalize[ReplacePart[mat, {1, 1, 1, 1}, 1]], 1];
  z = 2 Random[] - 1; r = Sqrt[1 - z^2];
  t = 2 Pi Random[];
  Graphics3D[{
    Opacity[0.5],
    Union[Flatten[ss]] /. s_Simplex → {
      Hue[Random[]],
      Polygon[mat.(# /. rule) & /@ List @@ s]
    }
  ],
  Boxed → False, ViewPoint → 3 {r Cos[t], r Sin[t], z}
]
```

```
b31 = Draw3D[del[B[Simplex[e0, e1, e2, e3]]]]
```

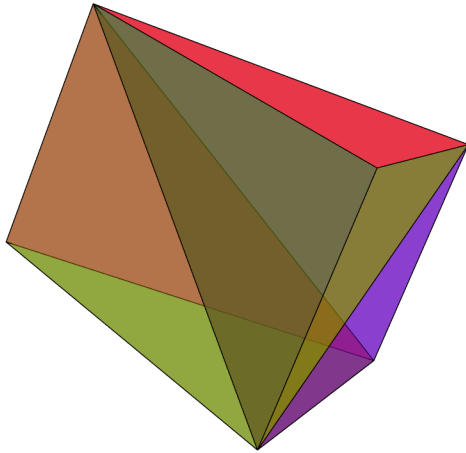


```
b32 = Draw3D[del[B[B[Simplex[e0, e1, e2, e3]]]]]
```



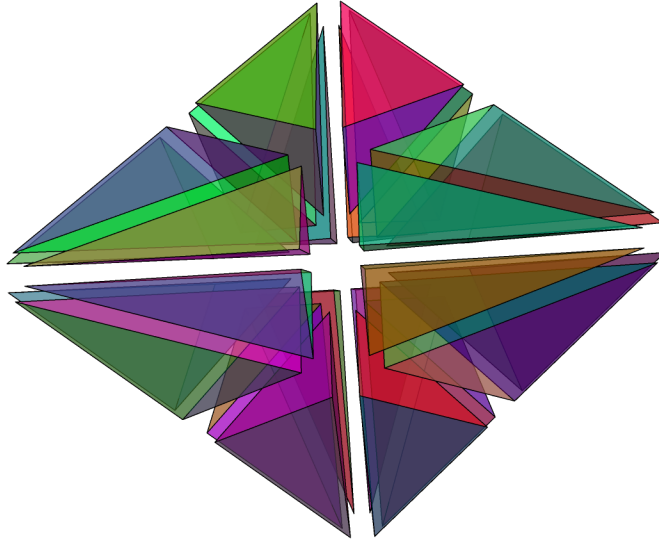
```
{f0, f1, f2} = {{1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}};  
{g0, g1, g2} = {{1, 0, 0, 1}, {0, 1, 0, 1}, {0, 0, 1, 1}};
```

```
p = Draw3D[del@{
  Simplex[f0, f1, f2, g2], Simplex[f0, f1, g1, g2], Simplex[f0, g0, g1, g2]
}
]
```

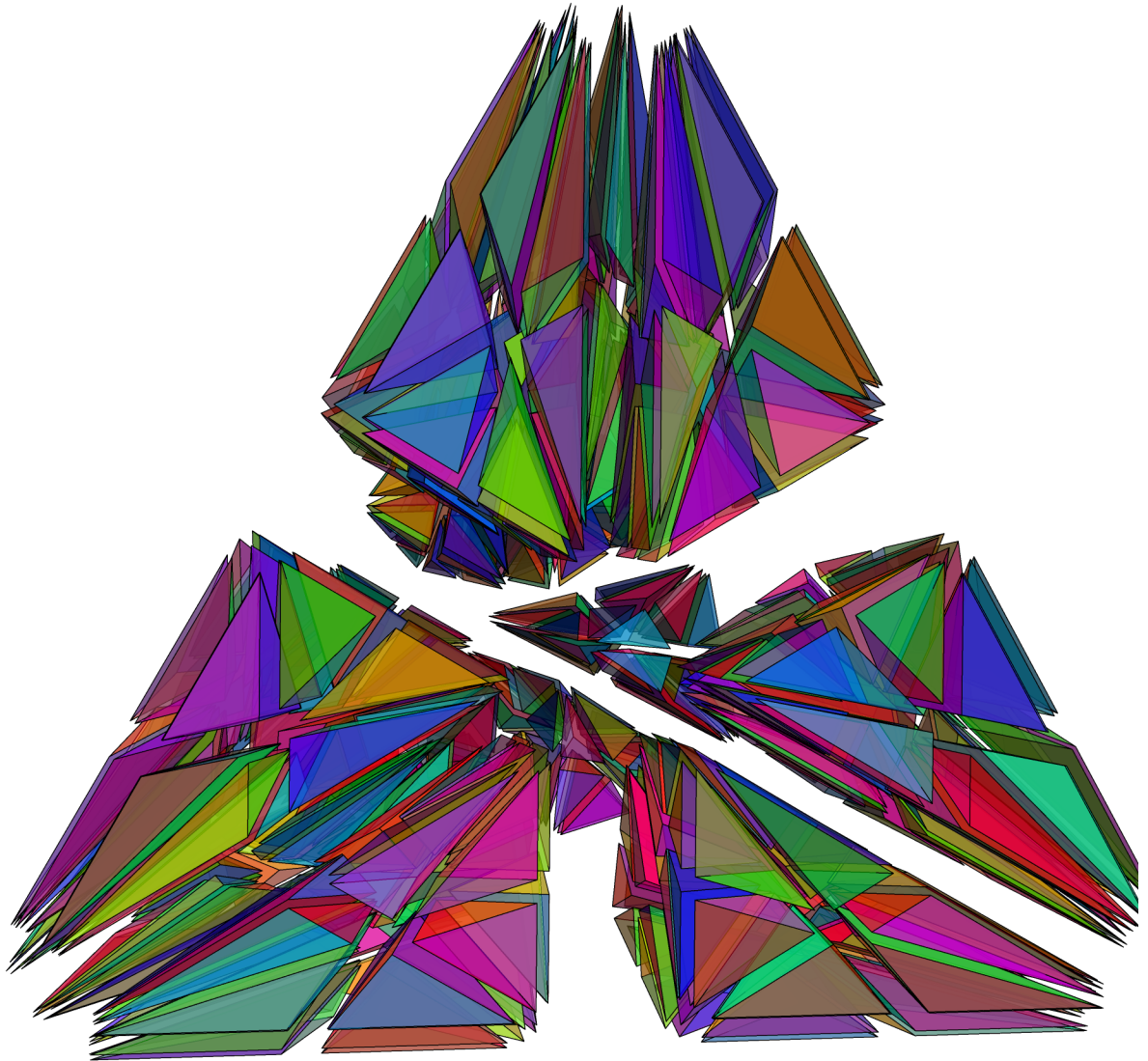


```
ShrinkTo[p_, s_Simplex] := Module[
  {b = (1 - p) Mean[List @@ s]},
  Expand[p # + b] & /@ s
];
ShrinkTo[p_, expr_] := expr /. s_Simplex => ShrinkTo[p, s]
```

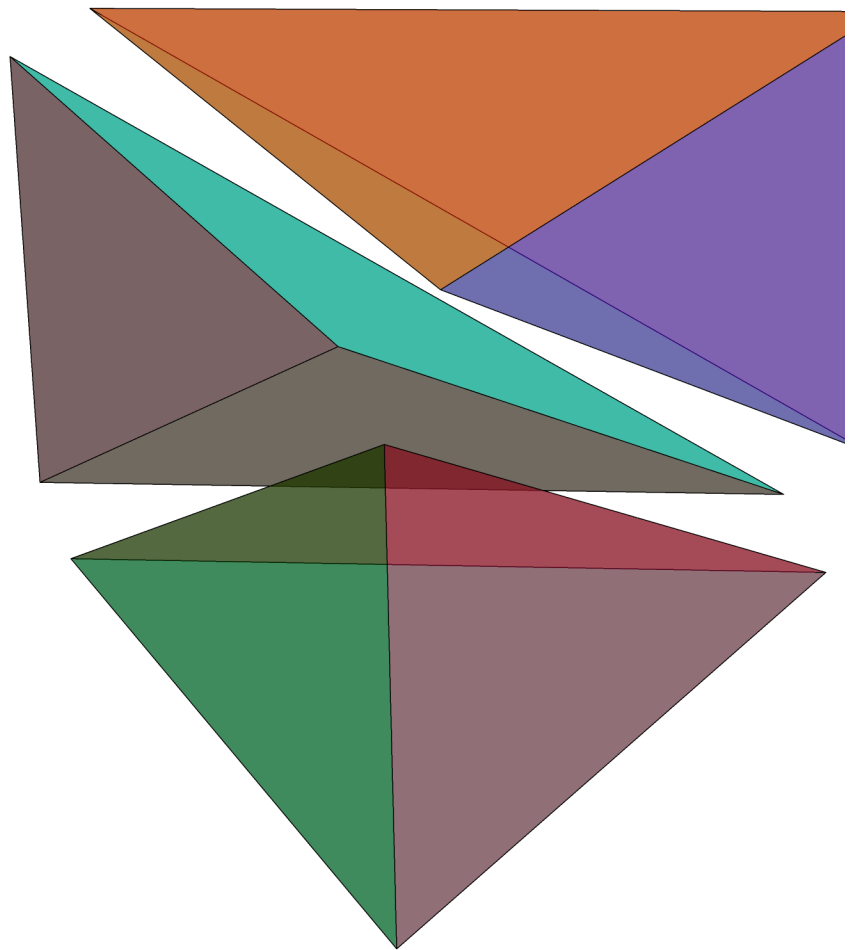
```
b31s = Draw3D[del[ShrinkTo[0.7, B[Simplex[e0, e1, e2, e3]]]]]
```




```
b32s = Draw3D[del[ShrinkTo[0.7, B[ShrinkTo[0.7, B[Simplex[e0, e1, e2, e3]]]]]]]
```



```
ps = Draw3D[del[ShrinkTo[0.7, {  
  Simplex[f0, f1, f2, g2], Simplex[f0, f1, g1, g2], Simplex[f0, g0, g1, g2]  
}]]
```



```
SetDirectory["C:/drorbn/AcademicPensieve/2010-06/"];
SaveImage[fname_String, img_] := Export[fname,
  ImageCrop[Rasterize[img, ImageSize -> {1600, 1600}]]
];
SaveImage["BarycentricSubdivision2D-1.png", b21];
SaveImage["BarycentricSubdivision2D-2.png", b22];
SaveImage["BarycentricSubdivision2D-3.png", b23];
SaveImage["BarycentricSubdivision3D-1.png", b31];
SaveImage["BarycentricSubdivision3D-1-Shrank.png", b31s];
SaveImage["BarycentricSubdivision3D-2.png", b32];
SaveImage["BarycentricSubdivision3D-2-Shrank.png", b32s];
SaveImage["Prizm.png", p];
SaveImage["Prizm-Shrank.png", ps];
```