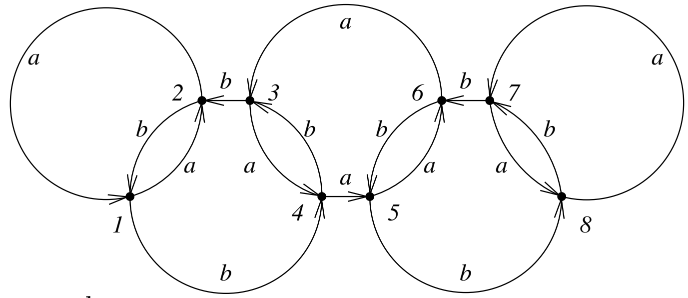
Homework Assignment 4

Problem 1. Let X be the "Olympic Rings" covering of the figure 8 space, 8^a_b, whose basepoint is taken to be at the quadrivalent vertex in its centre and whose fundamental group is the free group on two letter a and b: $G \coloneqq \pi_1(8^a_b) = F(a, b)$.



A. Describe the right G-set S corresponding to 8^a_b: it is a set with __ elements, and a and b act on it as the permutations ____ and ____.

B. Taking the basepoint x_1 of X to be the point marked as "1" on the right, write a set of generators for the image H of $\pi_1(X, x_1)$ within $G \coloneqq \pi_1(8^a_b)$.

C. Is H a normal subgroup of G?

Problem 2. Describe all the 2-sheeted and 3-sheeted connected coverings of the figure 8 space, 8^a_b. (Meanings, all the connected coverings that are 2 to 1 or 3 to 1).

Problem 3. Prove Corollary 11 from the Covering Spaces handout: If X is a connected covering of a nice space B (meaning, B is connected, locally connected and semi-locally simply connected) and $H \coloneqq p_\star\pi_1(X) < G \coloneqq \pi_1(B)$, then $\operatorname{Aut}(X) = N_G(H)/H$ where $N_G(H) \coloneqq \{g \in G \colon H = g^{-1}Hg\}$ is the normalizer of H in G.

Problem 4. Describe the universal covering space U of the space B which is the union of a 2-dimensional sphere and one of its diameter lines. (Don't say "it's the space of spelunkers" – you are expected to give a concrete description of U as some familiar space or as a simple subset of some familiar space).

Problem 5. If B is a nice space and U its universal cover, show that U is a covering of every connected covering X of B.