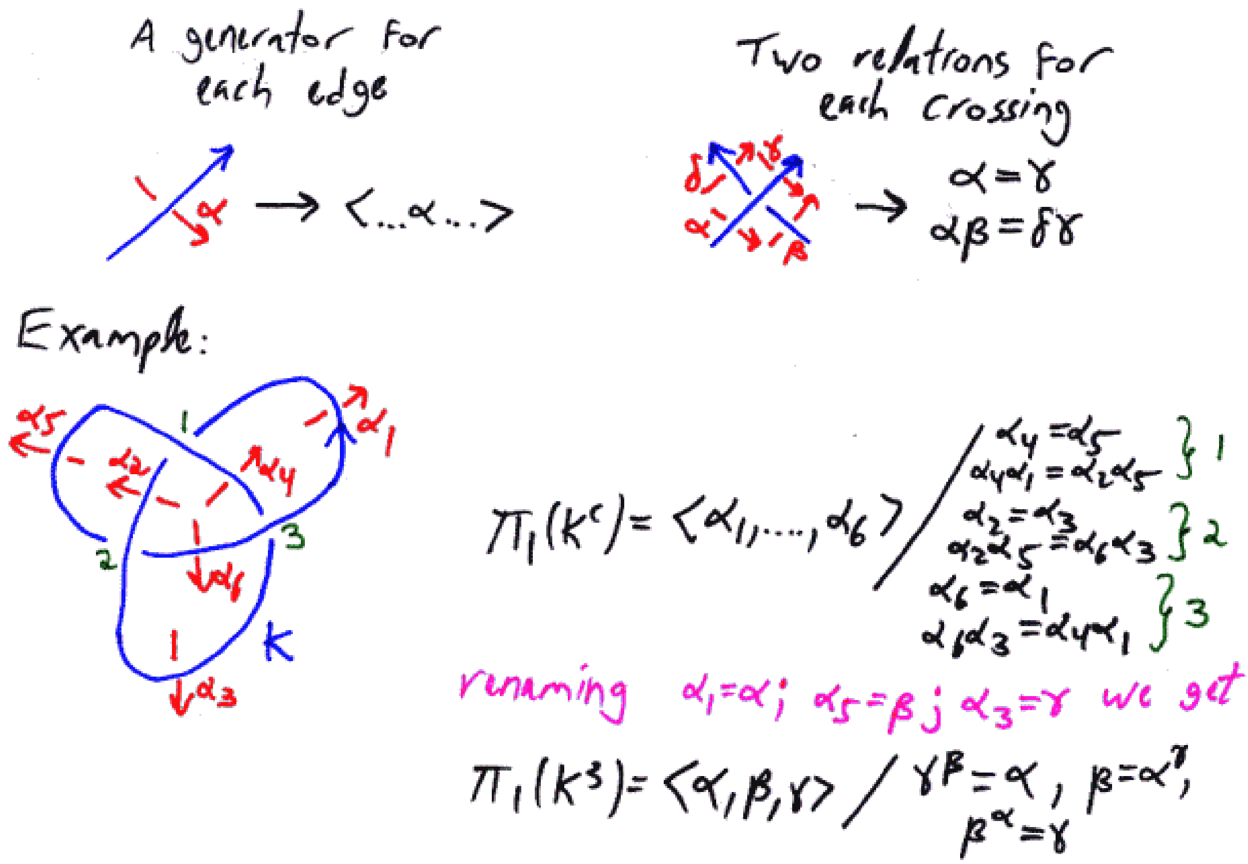
Homework Assignment 3

Problem 1 (40 points). Let K be a knot in $\mathbb{R}^3$ presented by a planar diagram D. With a massive use of van Kampen's theorem, show that the fundamental group of the complement of K has a presentation (the "Wirtinger presentation", as discussed in class) with one generator for each edge of D and two relations for each crossing of D, as indicated in the figure below.



Problem 2 (20 points). The trefoil knot above, whose fundamental group is $G_1 = \langle \alpha, \beta, \gamma \colon \alpha = \gamma^\beta,\ \beta = \alpha^\gamma,\ \gamma = \beta^\alpha \rangle$ is in fact the torus knot $T_{3/2}$, whose fundamental group, as computed in class, is $G_2 = \langle \lambda, \mu \colon \lambda^2 = \mu^3 \rangle$. Prevent the collapse of mathematics by showing that these two groups are isomorphic.