Homework Assignment 1

Problem 1. Write explicit formulas for the homotopy in https://drorbn.net/bbs/show?shot=25-1301-250106-142329.jpg between e and $f\bar{f}$ (and if that picture is wrong, fix it in your mind first). Your solution should take up 3 lines and must be of the form:

$$h(t,s) = \begin{cases} \text{formula 1} & \text{condition 1} \\ \text{formula 2} & \text{condition 2} \\ \text{formula 3} & \text{condition 3} \end{cases}$$

Problem 2. Prove the theorem which is implicit in the definition at https://drorbn.net/bbs/show?shot=25-1301-250107-162807.jpg. Namely, prove that if X is path-connected then the following are equivalent:

1. $\pi_1(X, x_0) = 0$ for some/any $x_0 \in X$.
2. If f_0 and F_1 are not-necessarily-closed paths that share their endpoints, namely $f_0(0) = f_1(0) = x_0$ and $f_0(1) = f_1(1) = x_1$, then they are homotopic via a homotopy that does not move these endpoints.
3. Any two maps $S^1 \to X$ are homotopic.

Problem 3. If X is path-connected, prove that the set of homotopy classes of maps $S^1 \to X$ can be put in a bijection with the set of conjugacy classes in the fundamental group of X.