Homework Assignment 7

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date

Tuesday, November 12, 2024 11:59 pm (Eastern Standard Time)

Late penalty

5% deducted per hour

Q1 (0 points)

Read sections 26, and 27 in Munkres' textbook (Topology, 2nd edition). Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read.

Also pre-read sections 51-55 of the same book.

Also, solve (but do not submit your solutions) problem 8 on page 171 and problem 5 on page 178 of the Munkres text.

Q2 (10 points)

(Munkres pp 171 ex 7)

Show that if $f\colon X \to Y$ is continuous where X is compact and Y is T_2, then f is closed. Meaning, if $A \subset X$ is closed, then so is $f(A) \subset Y$.

Q3 (20 points)

(Munkres pp 171 ex 9)

Let X and Y be topological spaces, and let $A \subset X$ and $B \subset Y$ be compact subsets thereof. Let $W \subset X \times Y$ be an open set such that $A \times B \subset W$. Show that there is an open rectangle $U \times V \subset X \times Y$ such that $A \times B \subset U \times V \subset W$.

Maybe you want to start with the case where X is a single point, and then reach to a single point and a little neighborhood thereof, and then do the full thing.

Q4 (20 points)

(Munkres pp 177 ex 2)

Let X be a metric space and let $A \subset X$ be non-empty. Let $d(x, A) := \inf_{a \in A} d(x, a)$ and let $U_\epsilon(A) := \{x : d(x, A) < \epsilon\}$.

(a) Show that $d(x, A) = 0$ iff $x \in \bar{A}$.

(b) Show that if A is compact, then $d(x, A) = d(x, a)$ for some $a \in A$.

(c) Show that $U_\epsilon(A)$ is the union of all the ϵ-balls whose centres lie in A.

(d) If A is compact and $U \supset A$ is open, show that for some $\epsilon > 0$ we have that $U_\epsilon(A) \subset U$.

(e) Find a counterexample to the result in (d), if A is not assumed to be compact.

Ready to submit?

→ Please ensure all pages are in order and rotated correctly before you submit

→ You will not be able to resubmit your work after the due date has passed.

Please wait...