Homework Assignment 6

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date

Tuesday, November 5, 2024 11:59 pm (Eastern Standard Time)

Late penalty

5% deducted per hour

Q1 (0 points)

Read sections 23, 24, 26, and 27 in Munkres' textbook (Topology, 2nd edition). Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read.

Also, solve (but do not submit your solutions) problems 1, 3, and 8acd on pages 157-158 and problems 1 and 6 on pages 170-171 of the Munkres text.

Q2 (10 points)

(Munkres pp 158 ex 2)

Let $f\colon S^1 = \{z \in \mathbb{C}\colon |z| = 1\} \to \mathbb{R}$ be a continuous function. Show that there exists $z \in S^1$ such that $f(z) = f(-z)$.

Q3 (10 points)

(Munkres pp 158 ex 8b)

If $A \subset X$ and A is path connected, is $\bar{A}$ always path connected too?

Q4 (10 points)

(Munkres pp 158 ex 10)

Show that an open connected subset U of $\mathbb{R}^n$ is path-connected.

Hint. Pick $x_0 \in U$ and show that the set of points in U that can be reached by a path from x_0 is clopen.

Q5 (10 points)

(Munkres pp 171 ex 2)

Let X be an uncountable set (e.g., $\mathbb{R}$).

(a) Show that the finite complement topology on X is compact.

(b) Is the countable-complement topology on X compact?

Q6 (10 points)

(Munkres pp 170 ex 4)

Show that every compact subspace A of a metric space M is closed and bounded (bounded means that the set of distances between elements of A is bounded).

Is the converse true? Namely, is it true that if A is closed and bounded in a metric space M then it is compact?

Q7 (10 points)

(Munkres pp 170 ex 5)

Show that if A and B are disjoint compact subsets of a Hausdorff space X, then there exists disjoint open sets U and V in X such that $A \subset U$ and $B \subset V$.

Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed.

Please wait...