Homework Assignment 5

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date
No due date yet

Late penalty
5% deducted per hour

Q1 (0 points)

Read sections 19 through 22 in Munkres' textbook (Topology, 2nd edition). Remember that reading math isn't like reading a novel! If you read a novel and miss a few details most likely you'll still understand the novel. But if you miss a few details in a math text, often you'll miss everything that follows. So reading math takes reading and rereading and rerereading and a lot of thought about what you've read. Also, preread sections 23 and 24, just to get a feel for the future.

Q2 (10 points)

(Munkres pp 118 ex 6)

Let $x_1, x_2, \ldots$ be a sequence of points in a product space $\prod X_\alpha$. Show that this sequence converges to the point x iff $\pi_\alpha(x_k) \to \pi_\alpha(x)$ for every α. (Note that if we don't specify the topology on a product space, we take it to be the cylinders topology).

Is the same fact true in the box topology?

Q3 (10 points)

(Munkres pp 118 ex 7)

Let $\mathbb{R}^\infty$ be the subset of $\mathbb{R}^\mathbb{N}$ consisting of the sequences that are almost always 0 - meaning, that are not zero only for finitely many indices. What is the closure of $\mathbb{R}^\infty$ in $\mathbb{R}^\mathbb{N}$ in the box and in the cylinders topology?

(Whatever is your answer, you need to prove it, of course).

Q4 (10 points)

(Munkres pp 126 ex 2)

Show that $\mathbb{R} \times \mathbb{R}$ in the dictionary order topology is metrizable.

Q5 (10 points)

(Munkres pp 126 ex 3, modified)

Let X be a metric space with metric d.

(a) Show that $d: X \times X \to \mathbb{R}$ is continuous.

(b) Show that the metric topology on X is the weakest (coarsest, smallest) topology on X relative to which $d: X \times X \to \mathbb{R}$ is continuous.

Q6 (10 points)

(Munkres pp 127 ex 5)

With the same notation as in Q3, what is the closure of $\mathbb{R}^\infty$ in $\mathbb{R}^\mathbb{N}$ using the uniform topology, defined by the metric $d(x, y) = \sup_k(\min(1, |x_k - y_k|))$.

Q7 (10 points)

(Munkres pp 127 ex 6)

With the same "uniform metric" as in the previous question, with some fixed $x = (x_1, x_2, \ldots)$ in $\mathbb{R}^\mathbb{N}$, and with some $0 < r < 1$, let $U(x, r) = \prod_k (x_k - r, x_k + r)$ (a product of intervals). Show that

(a) $U(x, r)$ is not equal to the ball $B_r(x)$.

(b) $U(x, r)$ is not even open in the uniform topology.

(c) $B_r(x) = \bigcup_{s<r} U(x, s)$.

Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed

Please wait...