Homework Assignment 1

Solve and submit your solutions of the following problems. Note that the late policy is very strict - you will lose 5% for each hour that you are late. In other words, please submit on time!

Due date

Tuesday, September 10, 2024 11:59 pm (Eastern Daylight Time)

Late penalty

5% deducted per hour

Q1 (10 points)

(Munkres pp20 ex1)

Let $f \colon A \to B$. Let $A_0 \subset A$ and let $B_0 \subset B$.

(a) Show that $A_0 \subset f^{-1}(f(A_0))$ and that equality holds if f is injective.

(b) Show that $f(f^{-1}(B_0)) \subset B_0$ and that equality holds if f is surjective.

Q2 (40 points)

(Munkres pp20-21 ex2)

Let $f: A \to B$ and let $A_i \subset A$ and $B_i \subset B$ for $i = 0, 1$. Show that f^{-1} preserves inclusions, unions, intersections, and differences of sets:

(a) $B_0 \subset B_1 \quad \implies \quad f^{-1}(B_0) \subset f^{-1}(B_1)$.

(b) $f^{-1}(B_0 \cup B_1) = f^{-1}(B_0) \cup f^{-1}(B_1)$.

(c) $f^{-1}(B_0 \cap B_1) = f^{-1}(B_0) \cap f^{-1}(B_1)$.

(d) $f^{-1}(B_0 \setminus B_1) = f^{-1}(B_0) \setminus f^{-1}(B_1)$.

Yet show that f only preserves inclusions and unions:

(e) $A_0 \subset A_1 \quad \implies \quad f(A_0) \subset f(A_1)$.

(f) $f(A_0 \cup A_1) = f(A_0) \cup f(A_1)$.

(g) $f(A_0 \cap A_1) \subset f(A_0) \cap f(A_1)$ and equality holds if f is injective.

(h) $f(A_0 \setminus A_1) \supset f(A_0) \setminus f(A_1)$ and equality holds if f is injective.

Q3 (20 points)

(Munkres pp21 ex3)

Show that (b), (c), (f), and (g) of the previous questions hold for arbitrary unions and intersections (finite or infinite).

Ready to submit?

- Please ensure all pages are in order and rotated correctly before you submit
- You will not be able to resubmit your work after the due date has passed

Submit pages for evaluation