

Pensieve header: Computing the Jones polynomial at Tsuda University.

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In[*]:= D1 = PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]
D2 = PD[X[4, 1, 5, 2], X[2, 5, 3, 6], X[6, 3, 1, 4]]
Out[*]=
PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]
Out[*]=
PD[X[4, 1, 5, 2], X[2, 5, 3, 6], X[6, 3, 1, 4]]

In[*]:= yusuke
Out[*]=
yusuke

In[*]:= yusuke = 7
Out[*]=
7

In[*]:= yusuke
Out[*]=
7

In[*]:= D1
Out[*]=
PD[X[1, 5, 2, 4], X[5, 3, 6, 2], X[3, 1, 4, 6]]

In[*]:= {a, a, b, b, c}
Out[*]=
{a, a, b, b, c}

In[*]:= {a, a, b, b, c} /. b -> yusuke
Out[*]=
{a, a, 7, 7, c}

In[*]:= {f[1], f[a], f[yusuke]} /. f[x_] -> x^2
Out[*]=
{1, a^2, 49}

In[*]:= t1 = D1 /. X[i_, j_, k_, L_] -> Ap[i, j] p[k, L] + Bp[i, L] p[j, k]
Out[*]=
PD[Ap[1, 5] p[2, 4] + Bp[1, 4] p[5, 2],
  Bp[3, 6] p[5, 2] + Ap[5, 3] p[6, 2], Bp[1, 4] p[3, 6] + Ap[3, 1] p[4, 6]]

In[*]:= t1
Out[*]=
PD[Ap[1, 5] p[2, 4] + Bp[1, 4] p[5, 2],
  Bp[3, 6] p[5, 2] + Ap[5, 3] p[6, 2], Bp[1, 4] p[3, 6] + Ap[3, 1] p[4, 6]]

In[*]:= Expand[(a + b) (c + d) (e + f)]
Out[*]=
a c e + b c e + a d e + b d e + a c f + b c f + a d f + b d f

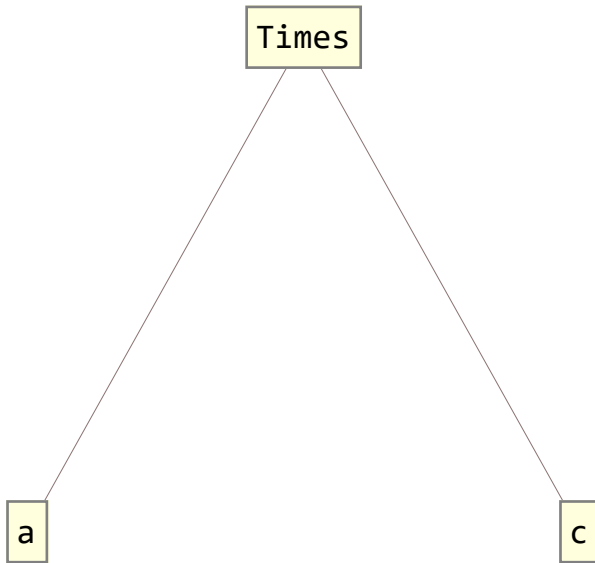
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```
In[*]:= FullForm[a c]
```

```
Out[*]//FullForm=  
Times[a, c]
```

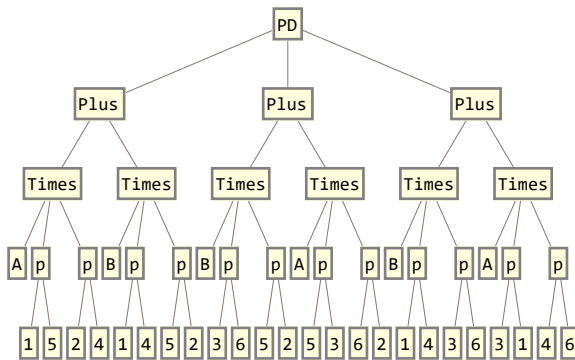
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In[*]:= TreeForm[a c]
```

```
Out[*]//TreeForm=
```



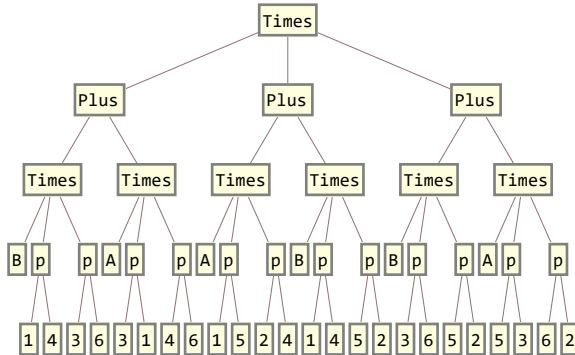
```
In[*]:= t1 // TreeForm
```

```
Out[*]//TreeForm=
```



```
In[*]:= TreeForm[t1 /. PD -> Times]
```

Out[*]//TreeForm=



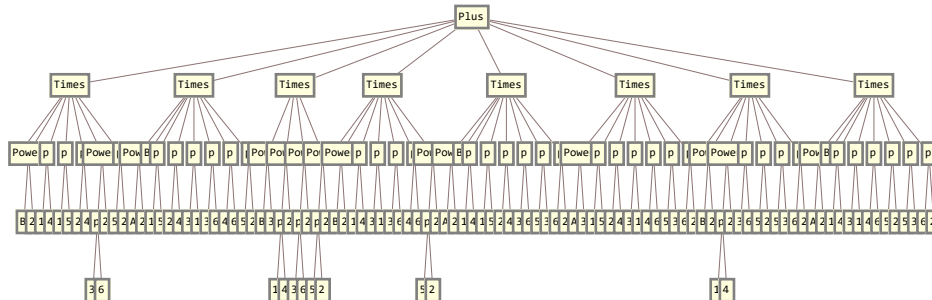
```
In[*]:= t2 = Expand[t1 /. PD -> Times]
```

Out[*]=

$$\begin{aligned}
 &A^2 B^2 p[1, 4] p[1, 5] p[2, 4] p[3, 6]^2 p[5, 2] + A^2 B p[1, 5] p[2, 4] p[3, 1] p[3, 6] p[4, 6] p[5, 2] + \\
 &B^3 p[1, 4]^2 p[3, 6]^2 p[5, 2]^2 + A B^2 p[1, 4] p[3, 1] p[3, 6] p[4, 6] p[5, 2]^2 + \\
 &A^2 B p[1, 4] p[1, 5] p[2, 4] p[3, 6] p[5, 3] p[6, 2] + \\
 &A^3 p[1, 5] p[2, 4] p[3, 1] p[4, 6] p[5, 3] p[6, 2] + \\
 &A B^2 p[1, 4]^2 p[3, 6] p[5, 2] p[5, 3] p[6, 2] + A^2 B p[1, 4] p[3, 1] p[4, 6] p[5, 2] p[5, 3] p[6, 2]
 \end{aligned}$$

```
In[*]:= t2 // TreeForm
```

Out[*]//TreeForm=



```
In[*]:= t3 = t2 /. {p[i_, j_] p[j_, k_] -> p[i, k]}
```

Out[*]=

$$\begin{aligned}
 &A^2 B p[1, 4]^2 + A^3 p[2, 2] p[3, 3] + A^2 B p[3, 6]^2 + A B^2 p[1, 4]^2 p[3, 6]^2 + A^2 B p[5, 2]^2 + \\
 &A B^2 p[1, 4]^2 p[5, 2]^2 + A B^2 p[3, 6]^2 p[5, 2]^2 + B^3 p[1, 4]^2 p[3, 6]^2 p[5, 2]^2
 \end{aligned}$$

```
In[*]:= t4 = t3 /. {p[i_, i_] -> d, p[i_, j_]^2 -> d}
```

Out[*]=

$$3 A^2 B d + A^3 d^2 + 3 A B^2 d^2 + B^3 d^3$$

```
In[*]:= t5 = Expand[t4 /. {B -> 1/A, d -> -A^2 - A^-2}]
```

Out[*]=

$$-\frac{1}{A^9} + \frac{1}{A} + A^3 + A^7$$

In[*]:= **t6 = Simplify** $\left[(-A^3)^{-3} \frac{t5}{-A^2 - A^{-2}} /. A \rightarrow q^{-1/4}\right]$

Out[*]=
 $q + q^3 - q^4$

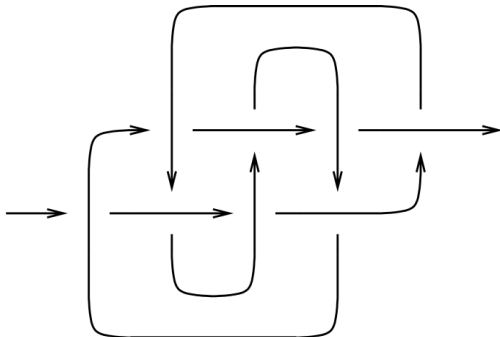
In[*]:= **J**[*pd_*, *w_*] := (
 $t1 = pd /. X[i_, j_, k_, l_] \Rightarrow Ap[i, j] p[k, l] + Bp[i, l] p[j, k];$
 $t2 = Expand[t1 /. PD \rightarrow Times];$
 $t3 = t2 //. \{p[i_, j_] p[j_, k_] \Rightarrow p[i, k], p[i_, j_] p[k_, j_] \Rightarrow p[i, k]\};$
 $t4 = t3 /. \{p[i_, i_] \Rightarrow d, p[i_, j_]^2 \Rightarrow d\};$
 $t5 = Expand[t4 /. \{B \rightarrow 1/A, d \rightarrow -A^2 - A^{-2}\}];$
 $Simplify\left[(-A^3)^{-w} \frac{t5}{-A^2 - A^{-2}} /. A \rightarrow q^{-1/4}\right]$
 $)$

In[*]:= **J**[**D1**, 3]

Out[*]=
 $q + q^3 - q^4$

In[*]:= **J**[**D2**, -3]

Out[*]=
 $\frac{-1 + q + q^3}{q^4}$

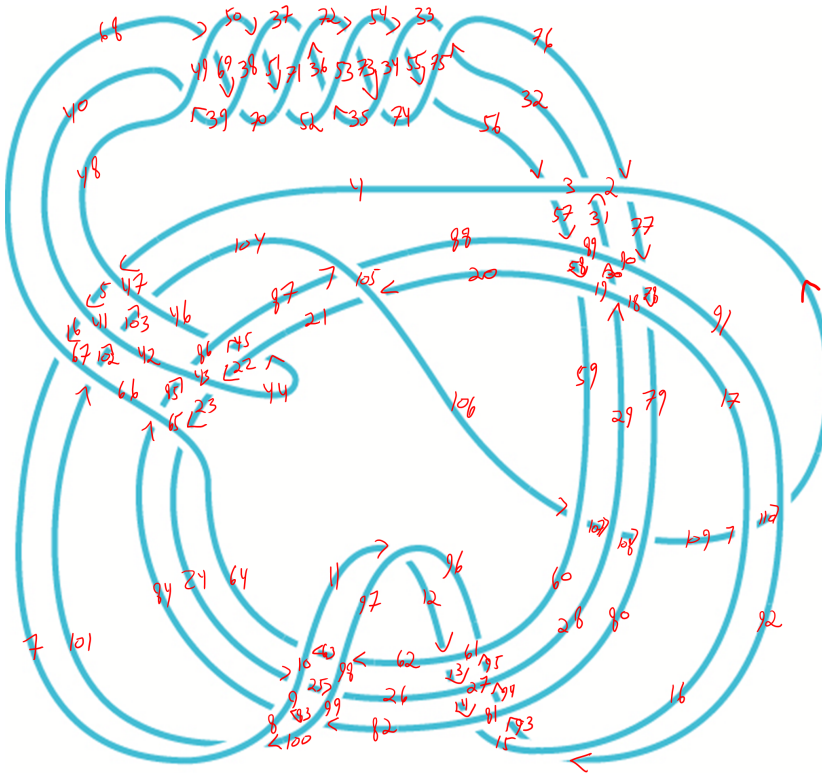


In[*]:= **K817 = PD**[**X**[6, 2, 7, 1], **X**[14, 8, 15, 7], **X**[8, 3, 9, 4], **X**[2, 13, 3, 14],
X[12, 5, 13, 6], **X**[4, 9, 5, 10], **X**[16, 12, 1, 11], **X**[10, 16, 11, 15]]

Out[*]=
PD[**X**[6, 2, 7, 1], **X**[14, 8, 15, 7], **X**[8, 3, 9, 4], **X**[2, 13, 3, 14],
X[12, 5, 13, 6], **X**[4, 9, 5, 10], **X**[16, 12, 1, 11], **X**[10, 16, 11, 15]]

In[*]:= **J**[**K817**, 0]

Out[*]=
 $7 + \frac{1}{q^4} - \frac{3}{q^3} + \frac{5}{q^2} - \frac{6}{q} - 6q + 5q^2 - 3q^3 + q^4$



```
In[ ] := PK = PD[
  X[4, 48, 5, 47], X[5, 40, 6, 41], X[6, 68, 7, 67], X[11, 97, 12, 96],
  X[12, 62, 13, 61], X[13, 26, 14, 27], X[14, 82, 15, 81], X[20, 105, 21, 106],
  X[22, 43, 23, 44], X[23, 65, 24, 64], X[24, 9, 25, 10], X[25, 99, 26, 98],
  X[29, 18, 30, 19], X[30, 90, 31, 89], X[31, 2, 32, 3], X[32, 76, 33, 75],
  X[35, 53, 36, 52], X[36, 72, 37, 71], X[39, 49, 40, 48], X[44, 21, 45, 22],
  X[45, 87, 46, 86], X[50, 38, 51, 37], X[51, 70, 52, 71], X[54, 34, 55, 33],
  X[55, 74, 56, 75], X[56, 4, 57, 3], X[57, 88, 58, 89], X[58, 20, 59, 19],
  X[62, 97, 63, 98], X[63, 11, 64, 10], X[68, 49, 69, 50], X[69, 39, 70, 38],
  X[72, 53, 73, 54], X[73, 35, 74, 34], X[76, 2, 77, 1], X[77, 90, 78, 91],
  X[78, 18, 79, 17], X[82, 99, 83, 100], X[83, 9, 84, 8], X[84, 65, 85, 66],
  X[85, 43, 86, 42], X[87, 105, 88, 104], X[92, 16, 93, 15], X[93, 80, 94, 81],
  X[94, 28, 95, 27], X[95, 60, 96, 61], X[100, 8, 101, 7], X[101, 66, 102, 67],
  X[102, 42, 103, 41], X[103, 46, 104, 47], X[106, 60, 107, 59], X[107, 28, 108, 29],
  X[108, 80, 109, 79], X[109, 16, 110, 17], X[110, 92, 1, 91]
];
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In[ ] := J[PK, w]
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Out[ ] :=
$Aborted
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