```
m_ ⋄ n_ := Min[m, n];
Kh2[K_] := Module[{pd, np, nm, p, S, a, ⋆, c, V, vp, vm, d, udeg, KC, v, dd, σ, Rank, B0, B1, dB0, db0, b1, Betti},
  pd = PD[K];
  np = Count[pd, X[i_, j_, k_, l_] /; j - l == 1 ∨ l - j > 1];
  nm = Count[pd, X[i_, j_, k_, l_] /; l - j == 1 ∨ j - l > 1];
  SetAttributes[p, Orderless];
  S[a_List] := S[a] = Times @@ ({List @@ pd, a}ᵀ /. {
           {X[i_, j_, k_, l_], 0} :> p[i, j]ᵢ⋄ⱼ p[k, l]ₖ⋄ₗ,
           {X[i_, j_, k_, l_], 1} :> p[i, l]ᵢ⋄ₗ p[j, k]ⱼ⋄ₖ,
           {x_X, ⋆} :> x}
        ) //. {
        p[i_, j_]ₘ_ p[j_, k_]ₙ_ :> p[i, k]ₘ⋄ₙ
       } //. {
       X[i_, j_, k_, l_] p[i_, j_]ₘ_ p[k_, l_]ₙ_ :> (cₘ cₙ → cₘ⋄ₙ),
       X[i_, j_, k_, l_] p[i_, l_]ₘ_ p[j_, k_]ₙ_ :> (cₘ⋄ₙ → cₘ cₙ)
      } //. p[__]ₘ̄_ :> cₘ;
  V[a_] := V[a] = List @@ Expand[S[a] /. c_x_ :> (vp_x + vm_x)];
  d[a_] := d[a] = S[a] /. {
      (c_x_ c_y_ → c_z_) * _. :> {vp_x vp_y → vp_z, vp_x vm_y → vm_z, vm_x vp_y → vm_z, vm_x vm_y → 0},
      (c_z_ → c_x_ c_y_) * _. :> {vp_z → vp_x vm_y + vm_x vp_y, vm_z → vm_x vm_y}
     };
  udeg[P_] := Exponent[P /. {v_a_ :> q^Total[a], vp_ → q, vm_ → q^-1}, q];
  KC[r_] := KC[r] = If[r < -nm || r > np, {},
     Join @@ (((v_#) V[#]) & /@ Permutations[Table[0, np - r] ~Join~ Table[1, r + nm]])
    ];
  KC[r_, deg_] := KC[r, deg] = Cases[KC[r], u_ /; udeg[u] - 2 nm + np == deg];
  dd[expr_] := Expand[expr] /. s_ * v_a_ :> Expand[σ = 1;
      Sum[
       If[a[[i]] == 0, σ * v_ReplacePart[a,1,i] * s /. d[List @@ ReplacePart[a, ⋆, i]], σ *= -1;
        0], {i, Length[a]}
      ]
     ];
  Rank[r_, deg_] := Rank[r, deg] = (
     B0 = KC[r, deg];
     B1 = KC[r + 1, deg];
     If[B0 == {} ∨ B1 == {}, 0,
      dB0 = dd[B0];
      MatrixRank[Table[Coefficient[db0, b1], {db0, dB0}, {b1, B1}]]
     ]
    );
  Betti[r_, deg_] := Length[KC[r, deg]] - Rank[r, deg] - Rank[r - 1, deg];
  Sum[
   t^r q^deg Betti[r, deg],
   {r, -nm, np},
   {deg, Union[udeg /@ KC[r]] - 2 nm + np}
  ]
]
```