```
DeclareGroup[S_k_] := Module[{α, β, e, γs},
  Clear[G, n, g, ι, m, inv];
  G = PermutationCycles /@ (Permutations@Range@k);
  n = Length[G];
  Do[g[α] = e = G[[α]]; ι[e] = α, {α, n}];
  m[] = ι[Cycles[{}]];
  Do[m[α, β] = ι[g[α]~PermutationProduct~g[β]],
    {α, n}, {β, n}];
  m[α_] := α; m[α_, β_, γs__] := m[m[α, β], γs];
  Do[inv[α] = ι[InversePermutation[g[α]]], {α, n}]
 ]
```