First class on Friday September 10. Reading week is after Lecture 24. Regrets in blue, gaps in red.

Topics in Algebraic Topology I: Algebraic Knot Theory and Computation.

The destination will be "a poly-time computable strong knot invariant with good algebraic properties". But you will be taking the course for the journey, not for the destination: What are knots and what are some of the problems around them? Why care about "invariants with good algebraic properties"? What is the "Yang-Baxter equation"? What are "virtual tangles"? What are "Hopf algebras"? Why would a topologist care about computations in Heisenberg algebras more than most physicists? How does Gaussian integration, and how do Feynman diagrams, arise in pure algebra? What is the "Drinfel'd Double Procedure"? Are we there yet?

The professor for this class does not believe anything that he does unless it is coded and the code runs. A useful life skill you will learn here is that even the incredibly abstract can become a computer program, often with no loss to its beauty.

Lecture 1. Course Introduction.

Lecture 2. Knots and the Kauffman bracket. →R1.

Lecture 3. Mathematica and implementing the Kauffman bracket.

Following http://drorbn.net/syd3.

Lecture 4. A faster Kauffman bracket program.

Following http://drorbn.net/syd3, with `ThinPosition` from `WG.nb`.

Lecture 5. Tangles and planar algebras. →R2, →R3.

Lecture 6. Three basic problems: unknotting, genus, ribbon knots. Display a list of ribbon knots.

Lecture 7. Aside: the Seifert algorithm.

Lecture 8. Tangles and the three basic problems.

Lecture 9. The Yang-Baxter approach and the WG algebras.

Lecture 10. Implementation.

Lecture 11. Virtual tangles and rotational virtual tangles. Meta monoids.

Lecture 12. Quasi-triangular Hopf algebras and the Kerler algebra.

Lecture 13. Implementation.

Lecture 14. The Heisenberg algebra $\mathbb{H}$, hR_0, and the PBW principle.

Lecture 15. Generating functions and hm.

Lecture 16. Gaussians and compositions.

Lecture 17. Implementation, testing.

Lecture 18. Yang-Baxter. An aside on the harmonic oscillator.

Lecture 19. Γ calculus and the Alexander polynomial. →R4.

Lecture 20. hR_ϵ. Gaussian integration.

Lecture 21. Perturbation theory for Gaussian integration.

Lecture 22. Perturbation theory for Gaussian compositions.

Lecture 23. Implementation.

Lecture 24. The Rozansky-Overbay invariants.

Lecture 25. CU_0, QU_0, and the QU_0-calculus.

Lecture 26. Genus using QU_0.

Lecture 27. Fox-Milnor using QU_0?

Lecture 28. CU_ϵ, Wigner contractions, solvable approximation.

Lecture 29. OU tangles and the Drinfel'd Double procedure. A finite-dimensional example.

Lecture 30. QU_ϵ and P.

Lecture 31. The rest of the QU_ϵ structure.

Lecture 32. Implementation, verification, computation.

Lecture 33. QU_ϵ and genus.

Lecture 34. From QU_ϵ to hR_ϵ.

Lecture 35. ?

Lecture 36. ?

Regret 1. Khovanov homology.

Regret 2. Khovanov homology for tangles.

Regret 3. Other algebraic structures near knot theory: (monoidal) categories, braid groups. Also mention contraction (circuit) algebras, meta-monoids, meta-Hopf-algebras, quandles, ...

Regret 4. Oh there's so much more on the Alexander polynomial!!!