Past and Future

First class on Friday September 10. Reading week is after Lecture 24. Regrets in blue, gaps in red.

Topics in Algebraic Topology I: Algebraic Knot Theory and Computation.

The destination will be "a poly-time computable strong knot invariant with good algebraic properties". But you will be taking the course for the journey, not for the destination: What are knots and what are some of the problems around them? Why care about "invariants with good algebraic properties"? What is the "Yang-Baxter equation"? What are "virtual tangles"? What are "Hopf algebras"? Why would a topologist care about computations in Heisenberg algebras more than most physicists? How does Gaussian integration, and how do Feynman diagrams, arise in pure algebra? What is the "Drinfel'd Double Procedure"? Are we there yet?

The professor for this class does not believe anything that he does unless it is coded and the code runs. A useful life skill you will learn here is that even the incredibly abstract can become a computer program, often with no loss to its beauty.

Lecture 1. Course Introduction.

Lecture 2. Knots and the Kauffman bracket. →R1.

Lecture 3. Mathematica and implementing the Kauffman bracket.

Lecture 4. A faster Kauffman bracket program.

Lecture 5. Tangles and planar algebras. →R2, →R3.

Lecture 6. Three basic problems: unknotting, genus, ribbon knots. Display a list of ribbon knots.

Lecture 7. Aside: the Seifert algorithm.

Lecture 8. Tangles and the three basic problems.

Lecture 9. The Yang-Baxter approach and the WG algebras.

Lecture 10. π_1 and WG.

Lecture 11. Implementation.

Lecture 12. Virtual tangles and meta monoids.

Lecture 13. Rotational virtual tangles.

Lecture 14. The Kerler(?) algebra.

Lecture 15. Hopf algebras and the 4D Alexander algebra, from PD to RVK.

Lecture 16. The Heisenberg algebra $\mathbb{H}$, hR_0, and the PBW principle.

Lecture 17. The formula for hR_0, generating functions.

Lecture 18. The Weyl formula and hm.

Lecture 19. Gaussians and compositions.

Lecture 20. Gaussians and compositions (2).

Lecture 21. Implementation and testing of GDO.

Lecture 22. Contracting Gaussians: Algebra by the means of partial differential equations.

Lecture 23. Contracting Gaussians: Algebra by the means of partial differential equations (2).

Lecture 24. Γ calculus and the Alexander polynomial. →R4.

Lecture 25. Gaussian integration.

past above / future below

Lecture 26. hR_ϵ. Perturbation theory for Gaussian integration.

Lecture 27. Perturbation theory for Gaussian compositions.

Lecture 28. Implementation.

Lecture 29. The Rozansky-Overbay invariants.

Lecture 30. CU_0, QU_0, and the QU_0-calculus.

Lecture 31. Genus using QU_0.

Lecture 32. Fox-Milnor using QU_0?

Lecture 33. CU_ϵ, Wigner contractions, solvable approximation.

Lecture 34. OU tangles and the Drinfel'd Double procedure.

Lecture 35. QU_ϵ and P.

Lecture 36. The rest of the QU_ϵ structure.

Lecture 37. Implementation, verification, computation.

Lecture 38. QU_ϵ and genus.

Lecture 39. From QU_ϵ to hR_ϵ.

Lectures not given.

Lecture 40. the Taft algebra

Following Montgomery, Schneider, "Skew derivations of finite-dimensional algebras and actions of the double of the Taft Hopf algebra"? Following programs by Roland?

Regret 1. Khovanov homology.

Regret 2. Khovanov homology for tangles.

Regret 3. Other algebraic structures near knot theory: (monoidal) categories, braid groups. Also mention contraction (circuit) algebras, meta-monoids, meta-Hopf-algebras, quandles, ...

Regret 4. Oh there's so much more on the Alexander polynomial!!!