Homework Assignment 5

Problem 1. By whatever means you wish, compute $\sum_D \frac{1}{|\operatorname{Aut}(D)|}$ where D runs over all connected diagrams with exactly $m=10$ quadrivalent vertices (vertices that connect to 4 edges). For example, for $m=1$ this number is $\frac{1}{2^3}=\frac{1}{8}$, for $m=2$ it is $\frac{1}{2\cdot 4!}+\frac{1}{2^4}=\frac{1}{12}$ and for $m=3$ it is $\frac{11}{96}$.

Problem 2. By whatever means you wish, find the generating function for the multiplication map $m_k^{ij}\colon A_i\otimes A_j\to A_k$ of the associative algebra $A\coloneqq\langle\mathbf{a},\mathbf{x}\rangle/([\mathbf{a},\mathbf{x}]=\mathbf{x})$, relative to the ordered basis $(\mathbf{a},\mathbf{x})$.

Problem 3. By whatever means you wish, yet without using results derived in class for QU, prove that the element $R_{ij}\coloneqq\mathbb{e}^{\hbar\mathbf{b}_i\mathbf{a}_j+\hbar\mathbf{y}_i\mathbf{x}_j}$ of $(\mathcal{U}(sl^0_{2+})_i\otimes\mathcal{U}(sl^0_{2+})_j)[\![\hbar]\!]$, with sl^0_{2+} being sl^ϵ_{2+} at $\epsilon=0$, satisfies the Yang-Baxter equation (Reidemeister 3) in $\mathcal{U}(sl^0_{2+})^{\otimes 3}[\![\hbar]\!]$.