Fourth Meeting

June 29, 2017 1:44 PM

On board: Have: $f \sim g \Rightarrow f_* = g_*$, $X \sim Y \Rightarrow H_\cdot(X) \cong H_\cdot(Y)$, $H_n(D^p) = H_n(pt)$

Want. $H_n(S^n) = \mathbb{Z}$.

observe: $\underset{A}{\emptyset \to S^{n-1}} \longrightarrow \underset{X}{D^n} \longrightarrow \underset{X/A}{D^n/S^{n-1}} = S^n \to \emptyset$ is "short exact"

$A \xrightarrow{\alpha} B \xrightarrow{\beta} C$ exact at B means $\operatorname{im}\alpha = \ker\beta$

$A \xrightarrow{\alpha} B \to 0$ exact $\Rightarrow \alpha$ is onto; $0 \to A \xrightarrow{\alpha} B$ exact $\Rightarrow \alpha$ is 1-1;

$0 \to A \xrightarrow{\alpha} B \to 0$ exact $\Rightarrow \alpha$ is iso; $0 \to A \xrightarrow{\alpha} B \xrightarrow{\beta} C \to 0$ exact $\Rightarrow C = B/A$

Def $0 \to C_n(A) \longrightarrow C_n(X) \longrightarrow C_n(X,A) \longrightarrow 0$

"relative chains" $\Rightarrow H_n(X,A)$ "relative homology"

Thm If $0 \to A_\cdot \longrightarrow B_\cdot \longrightarrow C_\cdot \longrightarrow 0$ is a short exact sequece of chain complexes, then....

Example 1. $S^{p-1} \hookrightarrow D^p \longrightarrow (D^p, S^{p-1})$

2. $D^p_- \longrightarrow S^p \longrightarrow (S^p, D^p_-)$

Excision $A \subset X$, $V \subset A$ open w/ $\bar{V} \subset \operatorname{int} A \Rightarrow$

$$H_*(X-V, A-V) \xrightarrow{\sim} H_*(X,A)$$

Use to prove $H(S^p, D^p_-) \cong H(D^p, S^{p-1})$, hence complete $H^n(S^n) \cong H^{n-1}(S^{n-1})$ for $n \geq 2$

$\tilde{H}$ & basing the induction.