Pensieve header: Khovanov Homology, Day 3.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; ~~the Catalan numbers~~; ~~the Jones polynomial~~; ~~a more efficient Jones algorithm~~; ~~a riddle on spheres~~; Khovanov homology; Γ-calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; ~~an order-4 torus~~; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; ~~sound experiments~~; barycentric subdivisions; ~~some Peano curves~~; braid closures and Vogel's algorithm; ~~the insolubility of the quintic~~; phase portraits; the Mandelbrot set; shadows of the Cantor square; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi~~; ~~Hochschild homology of (some) coalgebras~~; ~~convolutions and image improvements~~; ~~the 8-5-3 milk jug problem~~; ~~a cow problem,~~ a permutations package.

An NCGE Challenge

Update the NCGE program to contain "backtracking information". Use it to find how to turn the lower face of a Rubik's cube by turning all but the lower face of that cube.

The Package Perm

Complete the package perm, with documentation and all. For Perm[5,2,3,1,4], etc, your package should know $\sigma\circ\tau$, σ^{-1}, $\sigma[\![i]\!]$, Pivot[σ], PermutationQ[σ], IdentityPermutation[n], it should interact well with Cycles, and its internals should be hidden. It should live in a file "Perm.m".

Quilt Plots

Write a sophisticated Mathematica package that implements QuiltPlot and DoubleQuiltPlot with many bells and whistles, as explained on November 20th.

Shadows of the Cantor Square

Draw approximations of the Cantor square C^2. Then rotate C^2 by an angle θ and project the rotation vertically, draw that projection (while explaining why it is what you think it is), and compute its measure. Perhaps also use Manipulate[..., {θ, 0, $\pi/2$}]?

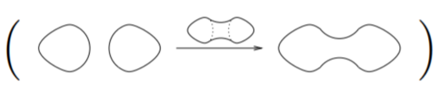
A Riddle

Everybody knows that the length of a smooth curve is the supremum of the lengths of its polygonal approximations. Everybody should know that the same is not true for smooth surfaces. Can you draw a smooth surface in $\mathbb{R}^3$ along with a triangulated approximation thereof whose area is vastly more? An appropriate animation may be even more convincing.

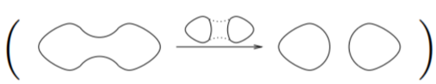
Khovanov: $K(L)$ is a chain complex of graded $\mathbb{Z}$-modules;
$V = \operatorname{span}\langle v_+, v_-\rangle$; $\deg v_\pm = \pm 1$; $q\dim V = q + q^{-1}$;

$$K(\bigcirc^k) = V^{\otimes k}; \quad K(\times) = \text{Flatten}\left(0 \to \underset{\text{height }0}{K()(\,)\{1\}} \to \underset{\text{height }1}{K(\asymp)\{2\}} \to 0\right);$$

$$K(\times) = \text{Flatten}\left(0 \to \underset{\text{height }-1}{K(\asymp)\{-2\}} \to \underset{\text{height }0}{K()(\,)\{-1\}} \to 0\right);$$



$\longrightarrow (V \otimes V \xrightarrow{m} V)$ $\quad m: \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0 \end{cases}$



$\longrightarrow (V \xrightarrow{\Delta} V \otimes V)$ $\quad \Delta: \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases}$

Khovanov Homology

Challenge: Implement Khovanov Homology as it is defined in the handout. Your implementation should fit in one page and should be good enough to show that $\mathrm{Kh}(5_1) \neq \mathrm{Kh}(10_{132})$.

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A = {{0, 0}, {1, 0}, {1, 0}, {0, 1}, {0, 0}, {1, 0}, {1, 0}, {0, 1}};
B = {{1, 0, 0, 0, -1, 0, 0, 0}, {0, 1, 1, 0, 0, -1, -1, 0}};
```