

Pensieve header: Three Peano curves.

? PeanoCurve

PeanoCurve[n] gives the line segments representing the n^{th} -step Peano curve. >

Peano Curve #1

`Simplify[a + z + 10 (t - 0.9) (-z)]`

$a + (10. - 10. t) z$

```

 $\phi_{0,a,-}[t_-] := \text{ReIm}[a];$ 
 $\phi_{n,a,z}[t_-] := \text{Which}[$ 
   $0 \leq t \leq 0.1, \text{ReIm}[a + 10 t z],$ 
   $0.1 \leq t \leq 0.3, \phi_{n-1,a+z,z/2}[5 (t - 0.1)],$ 
   $0.3 \leq t \leq 0.5, \phi_{n-1,a+z,i z/2}[5 (t - 0.3)],$ 
   $0.5 \leq t \leq 0.7, \phi_{n-1,a+z,-z/2}[5 (t - 0.5)],$ 
   $0.7 \leq t \leq 0.9, \phi_{n-1,a+z,-i z/2}[5 (t - 0.7)],$ 
   $0.9 \leq t \leq 1, \text{ReIm}[a + 10 (1 - t) z]$ 
 $];$ 
 $\phi_n[t_-] := \phi_{n,0,0.5(1+i)}[t_-];$ 

```

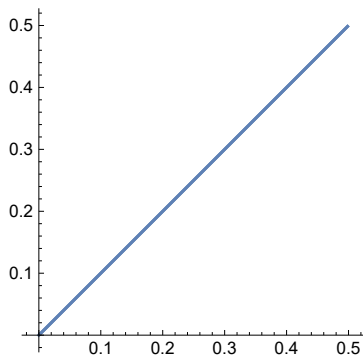
`Table[t → $\phi_1[t]$, {t, 0, 1, 0.05}]`

```

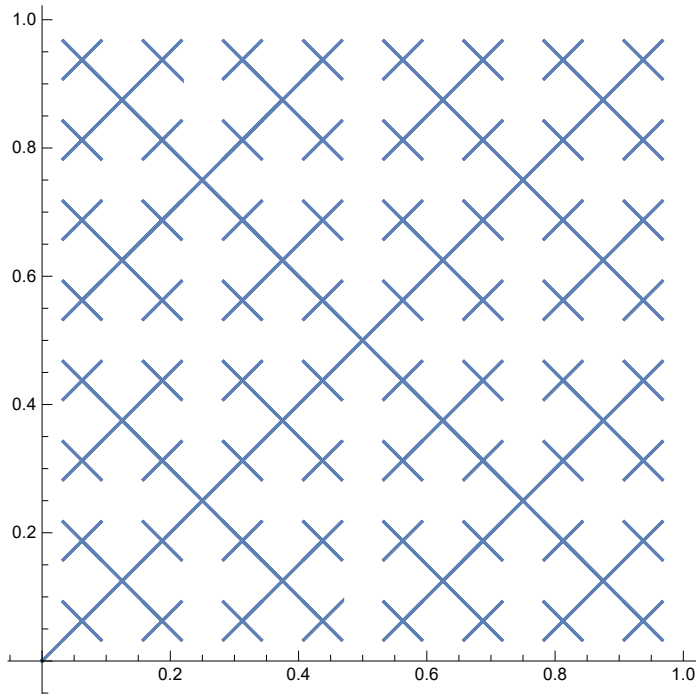
{0. → {0., 0.}, 0.05 → {0.25, 0.25}, 0.1 → {0.5, 0.5}, 0.15 → {0.5, 0.5}, 0.2 → {0.5, 0.5},
 0.25 → {0.5, 0.5}, 0.3 → {0.5, 0.5}, 0.35 → {0.5, 0.5}, 0.4 → {0.5, 0.5},
 0.45 → {0.5, 0.5}, 0.5 → {0.5, 0.5}, 0.55 → {0.5, 0.5}, 0.6 → {0.5, 0.5},
 0.65 → {0.5, 0.5}, 0.7 → {0.5, 0.5}, 0.75 → {0.5, 0.5}, 0.8 → {0.5, 0.5},
 0.85 → {0.5, 0.5}, 0.9 → {0.5, 0.5}, 0.95 → {0.25, 0.25}, 1. → {0., 0.}}

```

`ParametricPlot[$\phi_1[t]$, {t, 0, 1}, PlotRange → All]`



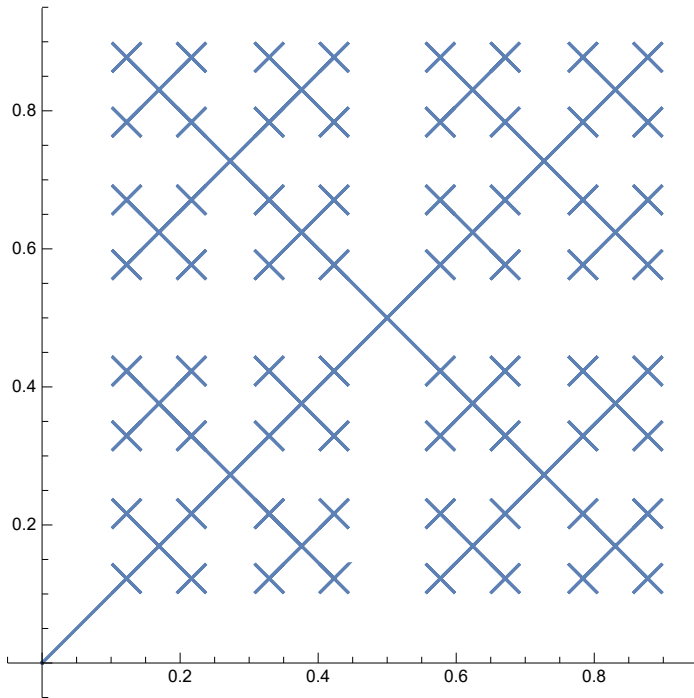
`ParametricPlot[$\phi_5[t]$, {t, 0, 1}, PlotRange -> All, PlotPoints -> 10000]`



```

 $\phi_{\theta,a,-}[t_-] := \text{ReIm}[a]$ ;
 $\phi_{n,a,z}[t_-] := \text{Which}[$ 
     $0 \leq t \leq 0.1, \text{ReIm}[a + 10 t z],$ 
     $0.1 \leq t \leq 0.3, \phi_{n-1,a+z,z/2.2}[5 (t - 0.1)],$ 
     $0.3 \leq t \leq 0.5, \phi_{n-1,a+z,i z/2.2}[5 (t - 0.3)],$ 
     $0.5 \leq t \leq 0.7, \phi_{n-1,a+z,-z/2.2}[5 (t - 0.5)],$ 
     $0.7 \leq t \leq 0.9, \phi_{n-1,a+z,-i z/2.2}[5 (t - 0.7)],$ 
     $0.9 \leq t \leq 1, \text{ReIm}[a + 10 (1 - t) z]$ 
];
 $\phi_n[t_-] := \phi_{n,0,0.5(1+i)}[t_-]$ ;
    
```

`ParametricPlot[$\phi_5[t]$, {t, 0, 1}, PlotRange -> All, PlotPoints -> 10000]`



```

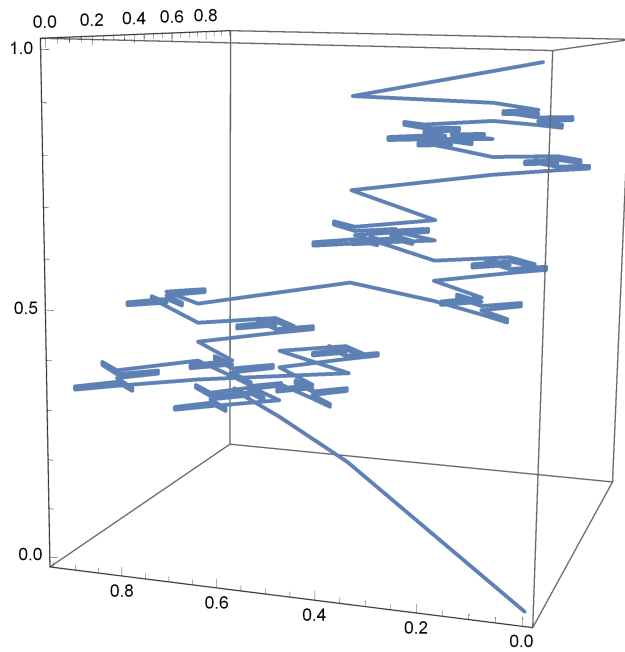
 $\phi_{0,a\_}[t\_]$  := ReIm[a];
 $\phi_{n,a,z}[t\_]$  := Which[
  0 ≤ t ≤ 0.1, ReIm[a + 10 t z],
  0.1 ≤ t ≤ 0.3,  $\phi_{n-1,a+z,z/2}[5(t - 0.1)]$ ,
  0.3 ≤ t ≤ 0.5,  $\phi_{n-1,a+z,i z/2}[5(t - 0.3)]$ ,
  0.5 ≤ t ≤ 0.7,  $\phi_{n-1,a+z,-z/2}[5(t - 0.5)]$ ,
  0.7 ≤ t ≤ 0.9,  $\phi_{n-1,a+z,-i z/2}[5(t - 0.7)]$ ,
  0.9 ≤ t ≤ 1, ReIm[a + 10 (1 - t) z]
];
 $\phi_n[t\_]$  :=  $\phi_{n,0,0.5(1+i)}[t]$ ;
 $\phi_{3n}[t\_]$  := Append[ $\phi_n[t]$ , t];

```

`Table[t -> $\phi_{3_1}[t]$, {t, 0, 1, 0.05}]`

- $\{0. \rightarrow \{0., 0., 0.\}, 0.05 \rightarrow \{0.25, 0.25, 0.05\}, 0.1 \rightarrow \{0.5, 0.5, 0.1\},$
- $0.15 \rightarrow \{0.5, 0.5, 0.15\}, 0.2 \rightarrow \{0.5, 0.5, 0.2\}, 0.25 \rightarrow \{0.5, 0.5, 0.25\},$
- $0.3 \rightarrow \{0.5, 0.5, 0.3\}, 0.35 \rightarrow \{0.5, 0.5, 0.35\}, 0.4 \rightarrow \{0.5, 0.5, 0.4\},$
- $0.45 \rightarrow \{0.5, 0.5, 0.45\}, 0.5 \rightarrow \{0.5, 0.5, 0.5\}, 0.55 \rightarrow \{0.5, 0.5, 0.55\},$
- $0.6 \rightarrow \{0.5, 0.5, 0.6\}, 0.65 \rightarrow \{0.5, 0.5, 0.65\}, 0.7 \rightarrow \{0.5, 0.5, 0.7\},$
- $0.75 \rightarrow \{0.5, 0.5, 0.75\}, 0.8 \rightarrow \{0.5, 0.5, 0.8\}, 0.85 \rightarrow \{0.5, 0.5, 0.85\},$
- $0.9 \rightarrow \{0.5, 0.5, 0.9\}, 0.95 \rightarrow \{0.25, 0.25, 0.95\}, 1. \rightarrow \{0., 0., 1.\}$

`ParametricPlot3D[$\phi_{3_4}[t]$, {t, 0, 1}, PlotRange -> All, PlotPoints -> 10000]`

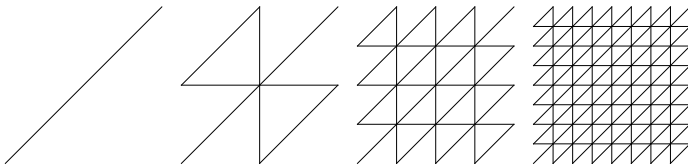


Peano Curve #2

```

Peano[0] = DLine[{{0, 0}, {1, 1}}];
Peano[n_] := Peano[n - 1] /. DLine[{{x0_, y0_}, {x1_, y1_}}] => {
  dx = x1 - x0; dy = y1 - y0;
  DLine[{{x0, y0}, {x0 + 1/2 dx, y0 + 1/2 dy}}],
  Line[{{x0 + 1/2 dx, y0 + 1/2 dy}, {x0 + 1/2 dx, y0}}],
  DLine[{{x0 + 1/2 dx, y0}, {x0 + dx, y0 + 1/2 dy}}],
  Line[{{x0 + dx, y0 + 1/2 dy}, {x0, y0 + 1/2 dy}}],
  DLine[{{x0, y0 + 1/2 dy}, {x0 + 1/2 dx, y0 + dy}}],
  Line[{{x0 + 1/2 dx, y0 + dy}, {x0 + 1/2 dx, y0 + 1/2 dy}}],
  DLine[{{x0 + 1/2 dx, y0 + 1/2 dy}, {x0 + dx, y0 + dy}}]
};
GraphicsRow@Table[Graphics[Peano[n] /. DLine -> Line], {n, 0, 3}]

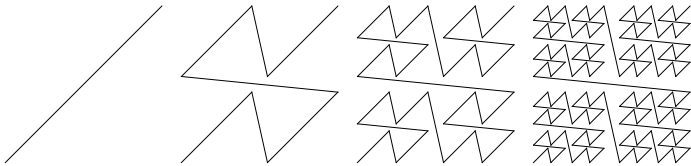
```



```

AlmostPeano[0] = DLine[{{0, 0}, {1, 1}}];
AlmostPeano[n_] := AlmostPeano[n - 1] /. DLine[{{x0_, y0_}, {x1_, y1_}}] => {
  dx = x1 - x0; dy = y1 - y0;
  DLine[{{x0, y0}, {x0 + 0.45 dx, y0 + 0.45 dy}}],
  Line[{{x0 + 0.45 dx, y0 + 0.45 dy}, {x0 + 0.55 dx, y0}}],
  DLine[{{x0 + 0.55 dx, y0}, {x0 + dx, y0 + 0.45 dy}}],
  Line[{{x0 + dx, y0 + 0.45 dy}, {x0, y0 + 0.55 dy}}],
  DLine[{{x0, y0 + 0.55 dy}, {x0 + 0.45 dx, y0 + dy}}],
  Line[{{x0 + 0.45 dx, y0 + dy}, {x0 + 0.55 dx, y0 + 0.55 dy}}],
  DLine[{{x0 + 0.55 dx, y0 + 0.55 dy}, {x0 + dx, y0 + dy}}]
};
GraphicsRow@Table[Graphics[AlmostPeano[n] /. DLine -> Line], {n, 0, 3}]

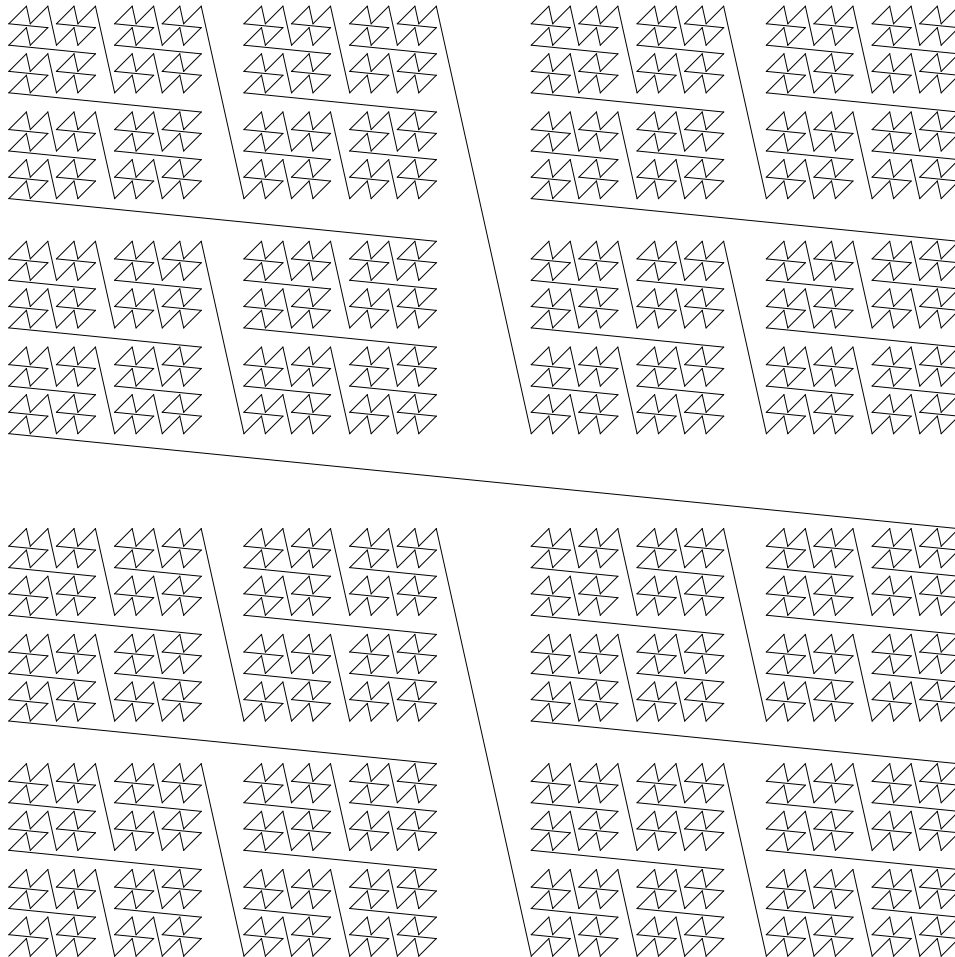
```



```

Graphics[AlmostPeano[5] /. DLine -> Line]

```



Peano Curve #3

```
 $\gamma[t\_List] := \text{Prepend}[$   
   $\text{FromDigits}[\{\#, 0\}, 2] \& /@ \text{Transpose}[\text{Partition}[t, 2]],$   
   $\text{FromDigits}[\{2t, 0\}, 3]$   
   $];$   
 $\text{Graphics3D}[$   
   $\text{Line}[\gamma /@ \text{Tuples}[\{0, 1\}, 14]],$   
   $\text{ViewPoint} \rightarrow \{-370.06, 733.124, 570.6\},$   
   $\text{ViewVertical} \rightarrow \{0.1792, 0.789508, -0.599889\}, \text{Boxed} \rightarrow \text{False}$   
   $] //$   
 $\text{Rasterize}$ 
```

