

Pensieve header: October 23: Some further Hochschild Homology.

Today. Some further Hochschild homology, then whatever you may suggest, then maybe EIWL 9-12, then, if we're kidding ourselves, Patterns.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers;~~ ~~the Catalan numbers;~~ ~~the Jones polynomial;~~ ~~a more efficient Jones algorithm;~~ ~~a riddle on spheres;~~ Khovanov homology; Γ -calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; **a Peano curve**; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; **the Mandelbrot set**; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi~~; **Hochschild homology of (some) coalgebras**; **convolutions and image improvements**.

An Image Manipulation Challenge

The image at <http://drorbn.net/bbs/show?shot=17-1750-171016-111042.jpg> is pathetic. Can you improve it? Whatever you do, should also work well with all other images at <http://drorbn.net/bbs/show.php?prefix=17-1750>.

Some Hochschild Homology

First see the image at <http://drorbn.net/AcademicPensieve/Classes/17-1750-ShamelessMathematica/index.html?im=171023-HomologyBBS.png>.

```
(*dn,k[ε_] := ε /. {xi/;i<k->xi, xi/;i==k->xk+xk+1, xi/;i>k->xi+1};*)
dn,k[ε_] := ε /. xi -> Which[i < k, xi, i == k, xk + xk+1, i > k, xi+1];
dn[ε_] := Expand@Sum[(-1)k dn,k[ε], {k, 0, n + 1}];
C0,p := If[p == 0, {1}, {}];
Cn,p := Cn,p = Union @@ Table[xnk Cn-1,p-k, {k, 0, p}];
```

?Switch

Switch[*expr*, *form*₁, *value*₁, *form*₂, *value*₂, ...] evaluates *expr*, then compares it with each of the *form*_{*i*} in turn, evaluating and returning the *value*_{*i*} corresponding to the first match found. >>

```
o[x_] := Switch[x, 2, 3, 4 | 5, 6, _, 7];
o /@ Range[10]
{7, 3, 7, 6, 6, 7, 7, 7, 7, 7}
```

C_{3,4}

```
{x14, x13 x2, x12 x22, x1 x23, x24, x13 x3, x12 x2 x3, x1 x22 x3, x23 x3, x12 x32, x1 x2 x32, x22 x32, x1 x33, x2 x33, x34}
```

d₃ /@ C_{3,4}

```
{-4 x13 x2 - 6 x12 x22 - 4 x1 x23, x13 x2 - 3 x12 x2 x3 - 3 x1 x22 x3, x12 x22 + 2 x12 x2 x3 - 2 x1 x2 x32, x1 x23 + 3 x1 x22 x3 + 3 x1 x2 x32,
x24 + 4 x23 x3 + 6 x22 x32 + 4 x2 x33 + x34, -x13 x4 - 3 x12 x2 x4 - 3 x1 x22 x4, -2 x1 x2 x3 x4, 2 x1 x2 x3 x4,
3 x22 x3 x4 + 3 x2 x32 x4 + x33 x4, -2 x12 x3 x4 - x12 x42 - 2 x1 x2 x42, -2 x1 x2 x3 x4, -2 x22 x3 x4 + 2 x2 x3 x42 + x32 x42,
-3 x1 x32 x4 - 3 x1 x3 x42 - x1 x43, -3 x2 x32 x4 - 3 x2 x3 x42 + x33 x4, -4 x33 x4 - 6 x32 x42 - 4 x3 x43}
```

d₄ /@ d₃ /@ C_{3,4}

```
{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

```

M_{n,p} := Transpose[Table[
  da = d_n[a];
  Table[Coefficient[da, b], {b, C_{n+1,p}},
  {a, C_{n,p}}
]];
rho_{n,p} := rho_{n,p} = MatrixRank[M_{n,p}];
beta_{n,p} := Length[C_{n,p}] - rho_{n,p} - rho_{n-1,p};

```

```
Table[beta_{n,p}, {n, 1, 5}, {p, 1, 5}] // MatrixForm // Timing
```

... Transpose: The first two levels of {} cannot be transposed.
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 ... Transpose: The first two levels of {} cannot be transposed.

... General: Further output of Transpose::nmtx will be suppressed during this calculation.

```

{0.46875, {
  1 - MatrixRank[Transpose[{}]] - MatrixRank[Transpose[{}]] - MatrixRank[Transpose[{}]] - MatrixRa
    0 0 0
    0 0 0
    0 0 0
    0 0 0
  }}

```

```
Table[beta_{n,p}, {n, 1, 5}, {p, 1, 5}] // MatrixForm // Timing
```

```

{0., {
  1 - MatrixRank[Transpose[{}]] - MatrixRank[Transpose[{}]] - MatrixRank[Transpose[{}]] - MatrixRank[Tr
    0 0 0
    0 0 0
    0 0 0
    0 0 0
  }}

```

```
Dimensions[m = M_{5,5}] // Timing
```

{0.1875, {252, 126}}

```
MatrixRank[m] // Timing
```

{0.015625, 86}