Pensieve header: October 23: Some further Hochschild Homology.

**Today.** Some further Hochschild homology, then whatever you may suggest, then maybe EIWL 9-12, then, if we're kidding ourselves, Patterns.

**Topics** (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; the Catalan numbers; ~~the Jones polynomial~~; ~~a more efficient Jones algorithm~~; ~~a riddle on spheres~~; Khovanov homology; Γ-calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish; ~~the Towers of Hanoi~~; Hochschild homology of (some) coalgebras; convolutions and image improvements.

## An Image Manipulation Challenge

The image at http://drorbn.net/bbs/show?shot=17-1750-171016-111042.jpg is pathetic. Can you improve it? Whatever you do, should also work well with all other images at http://drorbn.net/bbs/show.php?prefix=17-1750.

## Some Hochschild Homology

First see the image at http://drorbn.net/AcademicPensieve/Classes/17-1750-ShamelessMathematica/index.html?im=171023-HomologyBBS.png.

```
d_{n_,k_}[ℰ_] := ℰ /. {x_{i_} /; i < k :> x_i, x_{i_} /; i == k :> x_k + x_{k+1}, x_{i_} /; i > k :> x_{i+1}};
d_{n_}[ℰ_] := Expand@Sum[(-1)^k d_{n,k}[ℰ], {k, 0, n + 1}];
C_{0,d_} := If[d == 0, {1}, {}];
C_{n_,d_} := Union @@ Table[x_n^k C_{n-1,d-k}, {k, 0, d}];
```

```
C_{3,4}
```

$$\{x_1^4, x_1^3 x_2, x_1^2 x_2^2, x_1 x_2^3, x_2^4, x_1^3 x_3, x_1^2 x_2 x_3, x_1 x_2^2 x_3, x_2^3 x_3, x_1^2 x_3^2, x_1 x_2 x_3^2, x_2^2 x_3^2, x_1 x_3^3, x_2 x_3^3, x_3^4\}$$

```
d_3 /@ C_{3,4}
```

$$\{-4 x_1^3 x_2 - 6 x_1^2 x_2^2 - 4 x_1 x_2^3, x_1^3 x_2 - 3 x_1^2 x_2 x_3 - 3 x_1 x_2^2 x_3, x_1^2 x_2^2 + 2 x_1^2 x_2 x_3 - 2 x_1 x_2 x_3^2, x_1 x_2^3 + 3 x_1 x_2^2 x_3 + 3 x_1 x_2 x_3^2,$$
$$x_2^4 + 4 x_2^3 x_3 + 6 x_2^2 x_3^2 + 4 x_2 x_3^3 + x_3^4, -x_1^3 x_4 - 3 x_1^2 x_2 x_4 - 3 x_1 x_2^2 x_4, -2 x_1 x_2 x_3 x_4, 2 x_1 x_2 x_3 x_4,$$
$$3 x_2^2 x_3 x_4 + 3 x_2 x_3^2 x_4 + x_3^3 x_4, -2 x_1^2 x_3 x_4 - x_1^2 x_4^2 - 2 x_1 x_2 x_4^2, -2 x_1 x_2 x_3 x_4, -2 x_2^2 x_3 x_4 + 2 x_2 x_3 x_4^2 + x_3^2 x_4^2,$$
$$-3 x_1 x_3^2 x_4 - 3 x_1 x_3 x_4^2 - x_1 x_4^3, -3 x_2 x_3^2 x_4 - 3 x_2 x_3 x_4^2 + x_3 x_4^3, -4 x_3^3 x_4 - 6 x_3^2 x_4^2 - 4 x_3 x_4^3\}$$

```
d_4 /@ d_3 /@ C_{3,4}
```

$$\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

```
M_{n_,p_} := Table[Coefficient[d_n[a], b], {b, C_{n+1,p}}, {a, C_{n,p}}];
β_{n_,p_} := Length[NullSpace[M_{n,p}]] - MatrixRank[M_{n-1,p}];
```

```
Table[β_{n,p}, {n, 1, 5}, {p, 1, 5}] // MatrixForm // Timing
```

$$\{15.0469, \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\}$$