Today. A riddle on spheres, Charlene's project, Etienne's project, a more efficient Jones algorithm.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; ~~the Fibonacci numbers~~; the Catalan numbers; ~~the Jones polynomial~~; a more efficient Jones algorithm; a riddle on spheres; Khovanov homology; Γ-calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish.

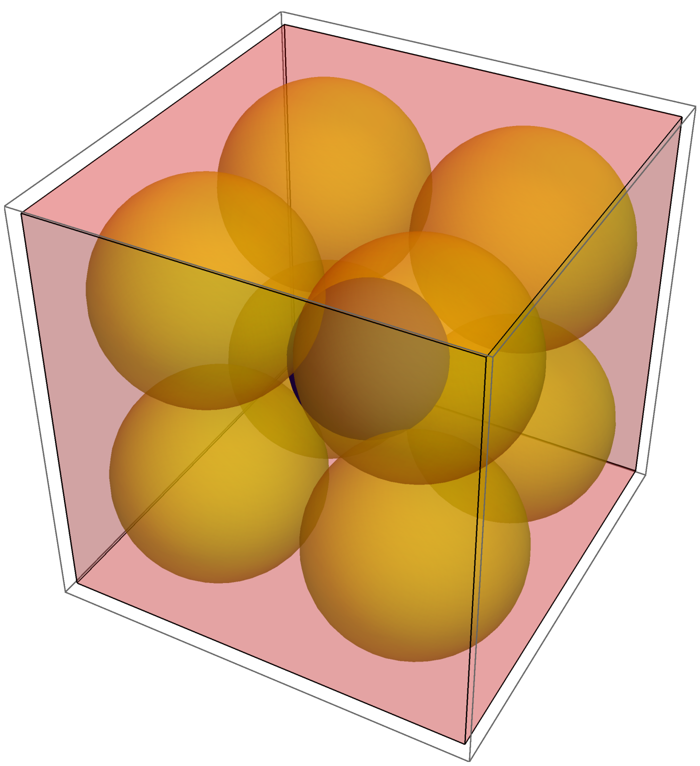
Pensieve header: October 6: A riddle on spheres.

A great riddle. 2^n yellow unit balls are centered at the vertices of the n-dimensional cube $\{-1, 1\}^n$. Let B_n be the largest blue ball centered at 0 bound by the yellow balls, and let C_n be the smallest red cube bounding the yellow balls. Compute $\lim_{n\to\infty} \frac{\mathrm{Vol}(B_n)}{\mathrm{Vol}(C_n)}$.

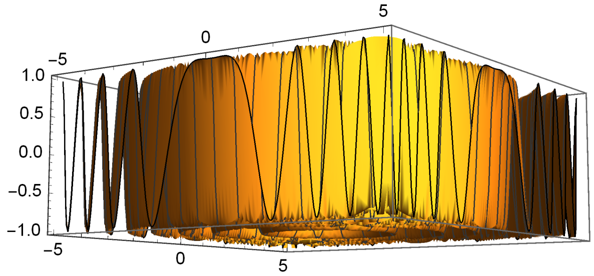
```
Tuples[{1, -1}, 3]
```

```
{{1, 1, 1}, {1, 1, -1}, {1, -1, 1}, {1, -1, -1},
 {-1, 1, 1}, {-1, 1, -1}, {-1, -1, 1}, {-1, -1, -1}}
```

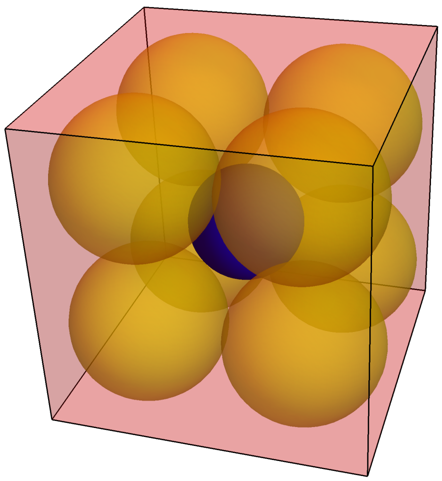
```
Graphics3D[{
  Yellow, Opacity[0.5], Table[Sphere[c, 1], {c, Tuples[{1, -1}, 3]}],
  Red, Opacity[0.2], Cuboid[{-2, -2, -2}, {2, 2, 2}],
  Blue, Opacity[1], Sphere[{0, 0, 0}, Sqrt[3] - 1]
 }]
```



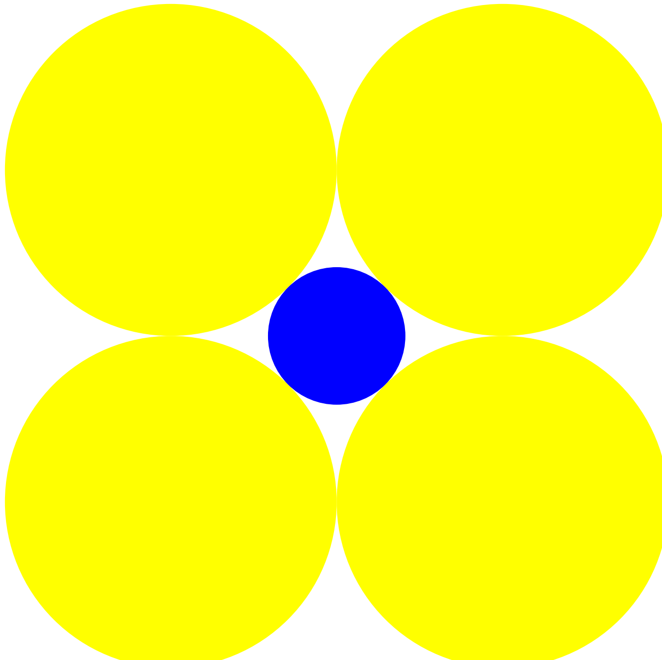
```
Plot3D[Cos[x^2 + y^2], {x, -5, 5}, {y, -5, 5}, PlotPoints → 50]
```




```
Graphics3D[{
  Yellow, Opacity[0.5], Table[Sphere[c, 1], {c, Tuples[{1, -1}, 3]}],
  Red, Opacity[0.2], Cuboid[{-2, -2, -2}, {2, 2, 2}],
  Blue, Opacity[1], Sphere[{0, 0, 0}, Sqrt[3] - 1]
 }, Boxed → False]
```



```
Graphics[{
  Yellow, Table[Disk[c, 1], {c, Tuples[{1, -1}, 2]}],
  Blue, Disk[{0, 0}, Sqrt[2] - 1]
 }]
```



volume of n dimensional ball

Result

$$\frac{2\,\pi^{n/2}\,r^n}{n\,\Gamma\left(\frac{n}{2}\right)} \approx \frac{2 \times 3.14159^{0.5\,n}\,r^n}{n\,\Gamma(0.5\,n)}$$

(assuming radius r)

$$r[\text{n\_}] := \frac{2\,\pi^{n/2}\left(\sqrt{n}\;-1\right)^n}{4^n\,n\,\Gamma\left(\frac{n}{2}\right)}$$

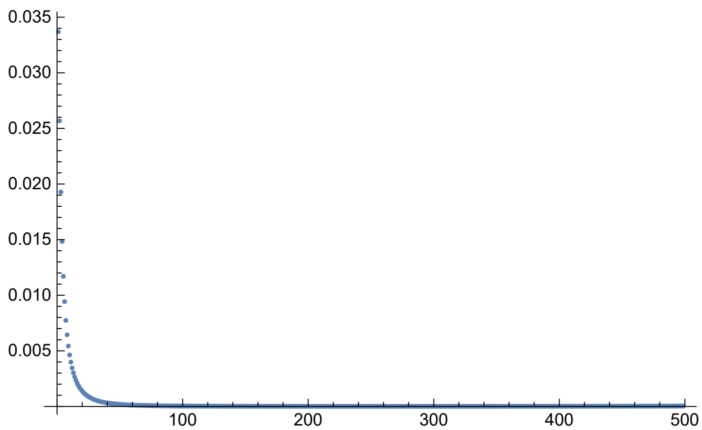
```
r[n]
```

$$\frac{2^{1-2\,n}\left(-1+\sqrt{n}\right)^n \pi^{n/2}}{n\,\text{Gamma}\left[\frac{n}{2}\right]}$$

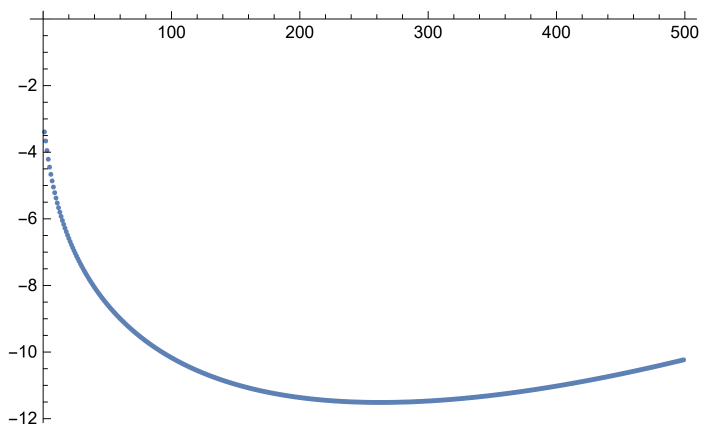
```
Table[r[n], {n, 2, 50}] // N
```

```
{0.0336883, 0.0256763, 0.0192766, 0.0148324, 0.0117012, 0.00942996, 0.00773623, 0.0064424,
 0.00543354, 0.00463295, 0.00398795, 0.00346144, 0.00302666, 0.00266395, 0.00235862,
 0.00209949, 0.00187795, 0.00168729, 0.0015222, 0.00137845, 0.00125265, 0.00114203,
 0.00104435, 0.000957722, 0.000880619, 0.000811745, 0.00075002, 0.000694528, 0.000644493,
 0.000599253, 0.000558243, 0.000520974, 0.000487026, 0.000456033, 0.000427678, 0.000401685,
 0.00037781, 0.00035584, 0.000335588, 0.000316888, 0.000299594, 0.000283573, 0.000268712,
 0.000254905, 0.000242061, 0.000230097, 0.000218937, 0.000208516, 0.000198772}
```

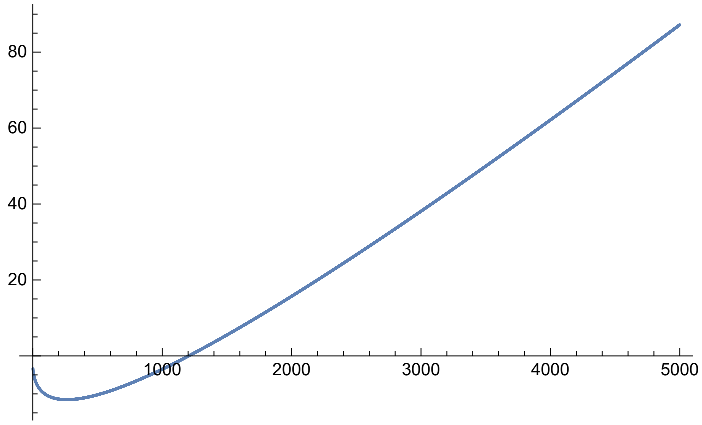
```
ListPlot[Table[r[n], {n, 2, 500}], PlotRange → All]
```




```
ListPlot[Table[Log[r[n]], {n, 2, 500}], PlotRange → All]
```



```
ListPlot[Table[Log[r[n]], {n, 2, 5000}]]
```




```
n = 2;
While[r[n] < 1, ++n];
n
```

```
1206
```

```
ListPlot[Table[Log[r[n]], {n, 1100, 1300}]]
```