Pensieve header: Sep 25: Further ways to compute Fibonacci.

Topics (in no particular order). Whatever you may suggest; whatever comes to my mind; the Fibonacci numbers; ~~the Jones polynomial~~; a more efficient Jones algorithm; a riddle on spheres; Khovanov homology; Γ-calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the Cantor Aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish.

Possible (Smallish) Projects

Get some neat things out of DBN: Classes: 2015-16: MAT 475 Problem Solving Seminar: Quiz 1:

```
Rasterize[
  Import[
    "http://drorbn.net/AcademicPensieve/Classes/16-475-ProblemSolving/Quiz-01.pdf"][[1]],
  ImageResolution → 120
 ] // ImageCrop
```

Name (Last, First): ______________________ Student ID: ______________________

Dror Bar-Natan: Classes: 2015-16: MAT 475 Problem Solving Seminar: http://drorbn.net/16-475

Quiz 1 on January 14, 2016: Search for a Pattern . You have 25 minutes to solve one of the two problems below. Please write on both sides of the page. **Good Luck!**

Problem 1. (Larson's 1.1.8; my estimate: medium) Prove that a list can be made of all the subsets of a nite set in such a way that (i) the empty set is rst in the list, (ii) each subset occurs exactly once, and (iii) each subset in the list is obtained from the preceding one either by adding one element or my removing one element.

Problem 2 (Larson's 1.1.9; my estimate: easy). Determine the number of odd coefficients in the expansion of $(x+y)^{1000}$.

Further ways to compute Fibonacci

The Naive Way

```
f0[0] = f0[1] = 1; f0[n_] := f0[n - 1] + f0[n - 2];
```

```
Table[Timing[n → f0[n]], {n, 20, 30}] // MatrixForm
```

0.	20 → 10946
0.03125	21 → 17711
0.046875	22 → 28657
0.0625	23 → 46368
0.109375	24 → 75025
0.15625	25 → 121393
0.265625	26 → 196418
0.46875	27 → 317811
0.90625	28 → 514229
1.4375	29 → 832040
2.53125	30 → 1346269

The Naive Way, Corrected

```
f1[0] = f1[1] = 1; f1[n_] := (f1[n] = f1[n - 1] + f1[n - 2]);
```

```
Timing[f1[100]]
```

{0., 573147844013817084101}

“prev”, “cur”, and “While”.

```
f2[n_] := (
  {k, prev, cur} = {1, 1, 1};
  While[k < n, {prev, cur} = {cur, prev + cur}; ++k];
  cur
 )
```

```
f2[100]
```

573147844013817084101

```
f3[n_] := Module[{k, prev, cur},
  {k, prev, cur} = {1, 1, 1};
  While[k < n, {prev, cur} = {cur, prev + cur}; ++k];
  cur
 ]
```

```
f3[40]
```

165580141

“prev”, “cur”, and “For”.

```
For[k = cur = prev = 1, k < 100, ++k, {prev, cur} = {cur, prev + cur}]; cur
```

573147844013817084101

"prev", "cur", and "Do".

```
{prev, cur} = {1, 1};
Do[{prev, cur} = {cur, prev + cur}, 99];
cur
```

```
573 147 844 013 817 084 101
```

A "While" loop for {f_1, f_2, ...} (using negative indices)

```
f6[n_] := Module[{fs = {1, 1}},
  While[Length[fs] ≤ n, fs = Append[fs, fs[[-1]] + fs[[-2]]]];
  Last[fs]
 ]
```

```
f6[100]
```

```
573 147 844 013 817 084 101
```

```
f7[n_] := Module[{fs = {1, 1}},
  While[Length[fs] ≤ n, fs = Append[fs, fs[[-1]] + fs[[-2]]]];
  fs
 ]
```

```
f7[20]
```

```
{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10 946}
```

```
f8[n_] := Module[{fs = {1, 1}},
  While[Length[fs] ≤ n, AppendTo[fs, fs[[-1]] + fs[[-2]]]];
  fs
 ]
```

```
f8[10]
```

```
{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89}
```

A "While" loop for {f_1, f_2, ...} (using "Total" and "Most")

```
f9[n_] := Module[{fs = {1, 1}},
  While[Length[fs] ≤ n, AppendTo[fs, 1 + Total@Drop[fs, -1]]];
  fs
 ]
```

```
f9[10]
```

```
{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89}
```

```
f10[n_] := Module[{fs = {1, 1}},
  While[Length[fs] ≤ n, AppendTo[fs, 1 + Total@Most@fs]];
  fs
 ]
```

```
f10[10]
{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89}
```

"Series" and $\frac{1}{1-x-x^2}$

```
Series[1/(1 - x - x^2), {x, 0, 10}]
```

$1 + x + 2 x^2 + 3 x^3 + 5 x^4 + 8 x^5 + 13 x^6 + 21 x^7 + 34 x^8 + 55 x^9 + 89 x^{10} + O[x]^{11}$

"SeriesCoefficient" and $\frac{1}{1-x-x^2}$

A Sum of Binomial Coefficients

Solve for an "explicit" formula, then use it.

Using "MatrixPower"

Using $f_{2n} = f_n^2 + f_{n-1}^2$ and $f_{2n+1} = f_n(f_{n+1} + f_{n-1})$

A "categorified" version (using lists)

A "categorified" version (using strings)

"ReplaceRepeated" on $\begin{pmatrix} n \\ f_{n-1} \\ f_n \end{pmatrix}$.

"NestWhile" on $\begin{pmatrix} n \\ f_{n-1} \\ f_n \end{pmatrix}$.

"Nest" on $\begin{pmatrix} f_{n-1} \\ f_n \end{pmatrix}$.

Other Items

Continue looking at Charlene's project? (Probably not; enough Fibonacci!)

A look at Etienne's project? (Maybe)