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## About This Class

**URL:** <http://drorbn.net/17-1750> and/or <http://www.math.toronto.edu/~drorbn/classes/17-1750/ShamelessMathematica/>.

**Agenda:** Sophisticated mathematics with Mathematica!

**Instructor:** [Dror Bar-Natan](#), [drorbn@math.toronto.edu](mailto:drorbn@math.toronto.edu) (for administrative matters only; math on email is slow and prone to misunderstandings, so I generally avoid it). Office: Bahen 6178, 416-946-5438.

**Classes:** Mondays 10-11 and Fridays 10-12 at Bahen 6183.

**Intended for.** *Graduate students.* While there are no formal prerequisites, a high level of mathematical sophistication will be assumed. Typically this would mean having taken a few core graduate classes here, or their parallels elsewhere.

**Office hours.** By appointment.

**Software.** By the time of the second class that you attend, you MUST arrange for yourself 24/7 access to [Mathematica](#), either by installing the \$150 student edition, or by whatever other means; otherwise please don't bother taking this class. There is a fair chance that the department will be able to help registered students with the purchase. Whatever you do, keep your receipts!

**Textbook.** [An Elementary Introduction to the Wolfram Language](#), by Stephen Wolfram.

**Course Description.** Yes, we all dream of the day we will prove that powerful theorem, whose beauty and sophistication will leave our colleagues breathless. It will, of course, be a product or pure thought, affirming that our intellect rises far, oh so far, beyond everybody else's. Obviously, no computers will be used. We are artists and philosophers, not technicians.

As a temporary measure I have learned to work with computers, and I plan to share what I have learned with you. For me, computer-assisted mathematics is a powered exoskeleton (seen Avatar? Iron Man?) for the brain. It's still my inner powers that everybody should admire, yet they reach much farther now that I've learned how to integrate them so tightly with the machine. Learn that too and reach far! I often use the platform "Mathematica" (though not only), and hence that's what I'll teach (though perhaps not only).



The Monday classes will be a systematic introduction to Mathematica following our [textbook](#).

The Friday classes will be divided into chapters, each about some (mathematical) real life problem that I have at some point encountered and solved with computers. The typical chapter will start with a mathematical introduction (sometimes deep and meaningful in itself). I will then pose a computational problem, and challenge you to solve it better than the solution that I will present. Many (though not all) of the problems will involve algebraic computations in knot theory, as this is what I know best. There will also be graphics, and some interaction with the web and with  $T_{E}X$ .

**Possible Topics** (in no particular order). Whatever you may suggest; whatever comes to my mind; the Fibonacci numbers; the Jones polynomial; a more efficient Jones algorithm; a riddle on spheres; Khovanov homology;  $\Gamma$ -calculus; the Hopf fibration; Hilbert's 13th problem; non-commutative Gaussian elimination; free Lie algebras; the Baker-Campbell-Hausdorff formula; wacky numbers; an order 4 torus; the Schwarz Lantern; knot colourings; the Temperley-Lieb pairing; the dodecahedral link; sound experiments; barycentric subdivisions; a Peano curve; braid closures and Vogel's algorithm; the insolubility of the quintic; phase portraits; the Mandelbrot set; shadows of the

Cantor Aerogel; quilt plots; some image transformations; De Bruijn graphs; the Riemann series theorem; finite type invariants and the Willerton fish.

**The Final Grade.** Impress me! And if you impress me enough and convince me that you've learned something, your grade will be high. Throughout the class I will suggest ideas for projects, and you can come up with your own ideas too. One idea you can always follow is to "one up" something that was done in class: make it better, cleaner, meaner, prettier, more efficient, more general, something. I will expect you to submit 4-5 projects during the semester.

**Elegance and cleanliness count!**

Projects are to be submitted via email to my normal email address [drorbn@math.toronto.edu](mailto:drorbn@math.toronto.edu). Each project MUST be a single file (though it can be a zip file that expands to a whole directory structure). Filenames MUST be of the form "lastname\_firstname\_date\_title.ext", where "lastname" is your last name, "firstname" is your first name, "date" is the date of submission in the form yymmdd (so the first day of classes, for example, is 170908), "title" is a very short descriptive title, and "ext" is some filename extension such as "nb" or "zip".

By default all submissions will be made public at <http://drorbn.net/AcademicPensieve/Classes/17-1750-ShamelessMathematica/StudentProjects/>, though you may request that specific projects will be kept private.

**Class Photo.** To help us learn each other's names, I will take a class photo on Friday of the third week of classes. I will post the picture on the class' web site and you will be *required* to identify yourself in the picture. With your individual consent, I will also post your names on the picture page.

## Apologies.

- There are places where exoskeleta simply don't work. Try swimming with one.
- It's a huge pain to learn exoskeleta. You think "I'll just wear it and start punching". Oh no, the balance is completely different; at first you'll simply fall over.
- Exoskeleta are bulky, rough and unpleasant. There are sharp ends, even, no, especially on the inside. Air conditioning? Don't make me laugh.
- Try showing up to the conference dinner wearing your exoskeleton. Nobody will want to sit next to you. Deep inside, everybody will think that you no longer belong.
- If ever you get too used to wearing your exoskeleton, you'll find that you are pink and vulnerable when you are not wearing it. If you want to remain tough, face the elements wearing only your sword.

Solution lines of the system

$$\begin{aligned}\dot{x} &= a_1 x^2 + \sqrt{2} b_1 x y + c_1 y^2, \\ \dot{y} &= a_2 x^2 + \sqrt{2} b_2 x y + c_2 y^2,\end{aligned}$$

with independent normal random  $a_i, b_i, c_i$ .

```
As := Table[As[iter___], {iter, 1, 100}];
Rasterize[GraphicsGrid[Table[As[i, 1, 5], As[j, 1, 5],
q = {a x^2 + sqrt(2) b x y + c y^2, a x^2 + sqrt(2) b x y + c y^2} /. a | b | c ->
RandomVariate[NormalDistribution[]];
Show[Join[
{VectorPlot[
{q, {x, -1, 1}, {y, -1, 1}, Frame -> None}},
Table[As[k, 1, 100],
eqns = Join[
Thread[{x'[t], y'[t]} == (q /. {x -> x[t], y -> y[t]}),
{x[0] == RandomReal[{-1, 1}], y[0] == RandomReal[{-1, 1]}];
sol = NDSolve[eqns, {x, y}, {t, -1, 1}];
ParametricPlot[
Evaluate[{x[t], y[t]} /. sol], {t, -1, 1},
PlotRange -> {{-1, 1}, {-1, 1}}, ColorFunction -> (Red &)]
]]], ImageSize -> 960]
```

