

Tuesday Jan 6, hour 1: Search for a Pattern

January-04-15 10:38 AM

make, bring textbook.

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1.1.1. Prove that a set of n (different) elements has exactly 2^n (different) subsets.

Too easy.

1.1.2. Let $S_{n,0}$, $S_{n,1}$, and $S_{n,2}$ denote the sum of every third element in the n th row of Pascal's Triangle, beginning on the left with the first element, the second element, and the third element respectively. Make a conjecture concerning the value of $S_{100,1}$.

done line - but no discussion of $\sqrt[3]{1}$.

1.1.3. Let x_1, x_2, x_3, \dots be a sequence of nonzero real numbers satisfying

$$x_n = \frac{x_{n-2}x_{n-1}}{2x_{n-2} - x_{n-1}}, \quad n = 3, 4, 5, \dots$$

Establish necessary and sufficient conditions on x_1 and x_2 for x_n to be an integer for infinitely many values of n .

1.1.4. Find positive numbers n and a_1, a_2, \dots, a_n such that $a_1 + \dots + a_n = 1000$ and the product $a_1 a_2 \dots a_n$ is as large as possible.

1.1.5. Let S be a set and $*$ be binary operation on S satisfying the two laws

$$x * x = x \quad \text{for all } x \text{ in } S,$$

$$(x * y) * z = (y * z) * x \quad \text{for all } x, y, z \text{ in } S.$$

Show that $x * y = y * x$ for all x, y in S .

$$\begin{aligned}
 ab &= \underset{x}{(a)} \underset{y}{(b)} \underset{z}{(ab)} = \underset{x}{(b)} \underset{y}{(ab)} \underset{z}{(a)} = \underset{||}{((ab)a)b} = \underset{||}{((ba)a)b} = \\
 &\underset{||}{(ab)a}b \quad \underset{||}{(ab)(ab)} \quad \underset{||}{(aa)b}b \\
 &\quad \quad \quad \underset{||}{(ab)b}b \\
 &\quad \quad \quad \underset{||}{(bb)a} \\
 &\quad \quad \quad ba
 \end{aligned}$$