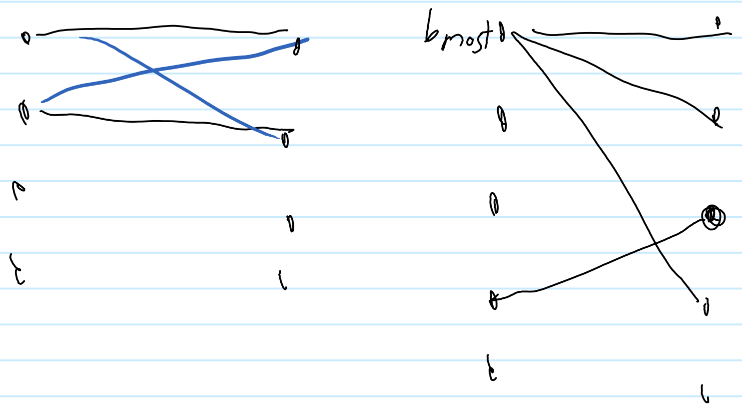
Thursday March 19, hours 29-30: Consider Extreme Cases

March-19-15 8:31 AM

1.11.1. Given a finite number of points in the plane, not all collinear, prove there is a straight line which passes through exactly two of them.

1.11.2. Let A be a set of $2n$ points in the plane, no three of which are collinear. Suppose that n of them are colored red and the remaining n blue. Prove or disprove: There are n closed straight line segments, no two with a point in common, such that the endpoints of each segment are points of A having different colors.

1.11.3. At a party, no boy dances with every girl, but each girl dances with at least one boy. Prove there are two couples bg and $b'g'$ which dance, whereas b does not dance with g' nor does g dance with b'.



1.11.4. Prove that the product of n successive integers is always divisible by $n!$.

1.11.5. Let $f(x)$ be a polynomial of degree n with real coefficients and such that $f(x) \geqslant 0$ for every real number x. Show that $f(x) + f'(x) + \cdots + f^{(n)}(x) \geqslant 0$ for all real x. ($f^{(k)}(x)$ denotes the kth derivative of $f(x)$.)

1.11.7. Show that there exists a rational number, c/d, with $d < 100$, such that

$$\left\llbracket k\frac{c}{d} \right\rrbracket = \left\llbracket k\frac{73}{100} \right\rrbracket \qquad \text{for} \quad k = 1, 2, 3, \ldots, 99.$$

Something about the Smith form.

3.311. Prove that there are an infinite number of primes of the form $6n-1$.

3.3.28.

(a) Suppose there are only a finite number of primes of the form $6n-1$; call them $p_1, \ldots, p_k$. Reach a contradiction by considering $N = (p_1 \cdots p_k)^2 - 1$.

(b) Prove that there are an infinite number of primes of the form $4n-1$.

Prove that if A is a clopen subset of [0,1] which contains 0, then A=[0,1].