Thursday Jan 8, hours 2-3: Quiz 1, Search for a Pattern

January-08-15 8:51 AM

Quiz.

on board during quiz:

Next Quiz: rest of 1.1, something/all from 1.2.

Pascal's triangle to row 6.

Problem 1.1.7 to row 11.

1.1.6. Beginning with 2 and 7, the sequence 2, 7, 1, 4, 7, 4, 2, 8, . . . is constructed by multiplying successive pairs of its members and adjoining the result as the next one or two members of the sequence, depending on whether the product is a one- or a two-digit number. Prove that the digit 6 appears an infinite number of times in the sequence.

Solve on board.

1.1.7. Let S_1 denote the sequence of positive integers $1, 2, 3, 4, 5, 6, \ldots$, and define the sequence S_{n+1} in terms of S_n by adding 1 to those integers in S_n which are divisible by n. Thus, for example, S_2 is $2, 3, 4, 5, 6, 7, \ldots$, S_3 is $3, 3, 5, 5, 7, 7, \ldots$. Determine those integers n with the property that the first $n-1$ integers in S_n are n.

Solve on board

Back to Pascal's triangle, using $\zeta = e^{2\pi i/3} = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

1.1.4. Find positive numbers n and $a_1, a_2, \ldots, a_n$ such that $a_1 + \cdots + a_n = 1000$ and the product $a_1 a_2 \cdots a_n$ is as large as possible.

1.1.5. Let S be a set and $*$ be binary operation on S satisfying the two laws

$$x * x = x \qquad \text{for all } x \text{ in } S,$$

$$(x * y) * z = (y * z) * x \qquad \text{for all } x, y, z \text{ in } S.$$

Show that $x * y = y * x$ for all x, y in S.

$$ab = (\underset{x}{a}\underset{y}{b})(\underset{z}{ab}) = (\underset{x}{b}\underset{y}{(ab)})\underset{z}{a} = ((ab)a)b = ((\underset{x}{b}\underset{y}{a})\underset{z}{a})b =$$

$((ab)a)b$ $\quad$ $(ab)(ab)$ $\quad$ $((aa)b)b$

$$\begin{aligned} &= (ab)b \\ &= (bb)a \\ &= ba \end{aligned}$$