

Name (Last, First): \_\_\_\_\_

Student ID: \_\_\_\_\_

Dror Bar-Natan: Classes: 2014-15: MAT 475 Problem Solving Seminar:

<http://drorbn.net/15-475>

**Quiz 4** “Choose an Effective Notation”, February 3, 2015. You have 30 minutes to solve as much as you can of the following problems. Please write on both sides of the page. **Good Luck!**

**Problem 1** (Larson’s 1.5.1). A couple days ago in the morning it started snowing at a heavy and constant rate. A snowplow started out at 8:00AM. At 9:00AM, it had gone 2km. By 10:00AM, it had gone 3km. Assuming that the snowplow removes a constant volume of snow per hour, determine the time at which it started snowing.

**Problem 2** (Larson’s 1.5.10). A well known theorem asserts that a prime  $p > 2$  can be written as a sum of two perfect squares ( $p = m^2 + n^2$  with  $m$  and  $n$  integers) iff  $p$  is 1 mod 4. Assuming this, prove:

1. Every prime which is 1 mod 8 can be written as  $x^2 + 16y^2$ , with  $x$  and  $y$  integers.
2. Every prime which is 5 mod 8 can be written as  $(2x + y)^2 + 4y^2$ , with  $x$  and  $y$  integers.

**Problem 3** (no credit, yet the best solutions will be advertised). What is your favourite “Modify the Problem” or “Choose an Effective Notation” problem?