240Algebra1-141201 Hour 35: Proofs of the basic properties of dets

November-25-14 3:59 PM

Last class. Wed 9-10 here. (OH 3-4)
Last week's schedule on web.
Course evals: 47/112 Warn the unsuspecting!

Goal: $|(a_{11})| := a_{11}$ $\quad \left|\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & \cdots & a_{nn}\end{pmatrix}\right| := \sum_{j=1}^{n} (-1)^{1+j} a_{1j} |A_{\hat{1}\hat{j}}|$
satisfies
$|E^1_{i,j} A| = -|A|$ $\quad |E^2_{i,c} A| = c|A|$ $\quad |E^3_{i,j,c} A| = |A|$

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1. Linear in the first row.
   Pf. "a_{1j}" is linear in first row, and a lin comp. of lin. functionals is linear.
2. Linear in all rows / Multilinear in the rows.
3. Vanishes if the first two rows are equal.
4. Vanishes if two adjacent rows are equal.

done line

5. Switches sign if two adjacent rows are interchanged.
6. Switches sign whenever two rows are interchanged.
7. $E^2_{i,c}$ & $E^3_{i,j,c}$ behavior.

delegated to tutorial

Problem. For any $A \in M_{n\times n}(F)$, compute A^p.

Example: $\begin{pmatrix} 4 & 3 \\ -6 & -5\end{pmatrix}^{15} = ?$

[here $C^{-1}AC = D$, w/ $D = \begin{pmatrix} 1 & 0 \\ 0 & -2\end{pmatrix}$, $C = \begin{pmatrix} 1 & -1 \\ -1 & 2\end{pmatrix}$]

Brilliant idea: If $A = CDC^{-1}$ for some C & a diagonal D, all is easy.

Also delegated to tutorial: Fibonacci rabbits.