Read Along 2.1–2.3

Riddle Along $\boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{7}\ \boxed{8}\ \boxed{9}$

Two players alternate drawing cards from the above deck. The first player to have 3 cards that add up to 15, wins. Would you like to be the first to move or the second? (More on today's web, including a video link).
https://media.library.utoronto.ca/play.php?DJ6CPFxByy2J&id=8503&access=public
https://cmc.math.ca/home/videos/game-of-15-and-isomorphisms/

HW6 on web.

Term test discussion & return @ 9:50.

Today: Linear transformations, abstractly.

Reminder. $V, W/F$ $L: V \to W$ "linear transformation" if it preserves structure:

$$L\left(\sum \alpha_i u_i\right) = \sum \alpha_i L(u_i)$$

* $\mathcal{L}(V, W)$ is said is a vector space.
* The composition of l.t. is a l.t.
* Not commutative!
* A l.t. is determined by its values on a basis, and these values are arbitrary.

done line

Def V & W are isomorphic if $\exists$ l.t. $R: V \to W$ and $L: W \to V$ s.t. $L \circ R = I_V$ & $R \circ L = I_W$

Thm If V, W are f.d. over F, then $\dim V = \dim W$ iff V is isomorphic to W.

Corollary If $\dim V = n$ over F, V is isomorphic to F^n.

Two "mathematical structures" are "isomorphic" if there's a bijection (1-1 & onto corres.) between their elements which preserves all relevant relations.

Example plastic chess is iso. to ivory chess, but not to checkers.

Example The game of 15.

Pf of Thm & of corollary

target line.

Fix a l.t. $T: V \to W$

Def $N(T) = \ker T = \{v : Tv = 0\}$ "null space", "kernel".

$R(T) = \operatorname{im} T = \{Tv : v \in V\}$ "range", "image"

Prop/Def $N(T) \subset V$ is a subspace; $\text{nullity}(T) := \dim N(T)$

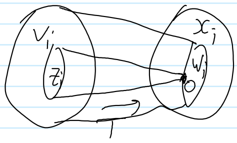
$R(T) \subset W$ is a subspace; $\text{rank}(T) := \dim R(T)$

Examples 0, I_V, $D : P_n(\mathbb{R}) \longrightarrow P_n(\mathbb{R})$

Thm 1 "the dimension theorem", "the rank-nullity Thm"

Given $T : V \to W$, $\underset{m}{\dim V} = \underset{r}{\text{rank}(T)} + \underset{n}{\text{nullity}(T)}$

Pf $(z_i)_1^n$ basis of $N(T)$, extend to $(z_i) \cup (v_i)$ a basis of V,



claim $w_i := T(v_i)$ are lin indep. in W pf....

claim w_i span $R(T)$ pf...