240Algebral-141001 Hours 11-12: Subspaces, linear combination

September-18-14 6:29 PM

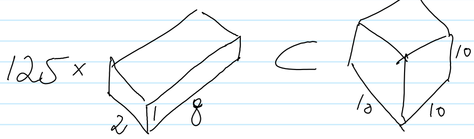
vitamins / HW3 handout.

HW2: by Friday 5PM @ labeled mailboxes, SS 1071 "Math Aid Centre".

Office Hours. Today 3-4.

Read Along 1.3-1.6

Riddle Along



Today: Subspaces, linear combinations, lin. independence.

Reminder <u>Def</u> $W \subset V$ is a "subspace" if it is a vector space with the operations it inherits from V.

<u>Thm</u> $W \subset V$ is a subspace iff it is non-empty & "closed under addition and under multiplication by a scalar"

<u>Examples</u> 1. $\{A \in M_{n\times n}(F) : A^t = A\}$

2. $\{A \in M_{n\times n}(F) : \operatorname{tr} A = 0\}$

3. If W_1 & W_2 are subspaces of V, The so is $W_1 \cap W_2$ (What about unions?)

Goal: Every V.S. has a "basis". So while we don't <u>have</u> to use coordinates, we can.

Wikipedia image

<u>Def</u> u is a l.c. of $u_1, \dots, u_n$ if $\exists a_i \in F$ s.t. $u = \sum a_i u_i$

<u>Examples</u> 1. Vitamins as in the handout

2. In $P_3(\mathbb{R})$, $2x^3 - 2x^2 + 12x - 6$ is a l.c. of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$

but $3x^3 - 2x^2 + 7x + 8$ isn't.

<u>Thm</u> If $\{u_i\} \subset V$ then $W = \operatorname{span}(u_i) := \{$all l.c. of the u_i's$\}$

is a subspace of V.

<u>Def</u> $S \subset V$ "generates" or "spans" V. (First requirement from "a basis")

<u>Examples</u> In $V = M_{2\times2}(\mathbb{R})$ $M_1 = \begin{pmatrix}1&0\\0&0\end{pmatrix}$ $M_2 = \begin{pmatrix}0&1\\0&0\end{pmatrix}$ $M_3 = \begin{pmatrix}0&0\\1&0\end{pmatrix}$ done
$M_4 = \begin{pmatrix}0&0\\0&1\end{pmatrix}$ $N_1 = \begin{pmatrix}0&1\\1&1\end{pmatrix}$ . . . $N_4 = \begin{pmatrix}1&1\\1&0\end{pmatrix}$, Then
$M_1 \dots M_4$ & $N_1 \dots N_4$ generate V, but
$M_1 \dots M_3$ & $N_1 \dots N_3$ do not.

Aside: If $S_1 \subset \mathrm{Span}(S_2)$ then $\mathrm{Span}(S_1) \subset \mathrm{Span}(S_2)$

$\left(\underset{\begin{pmatrix}0&1\\1&1\end{pmatrix}}{\overset{\shortparallel}{N_1}}, \underset{\begin{pmatrix}1&0\\1&1\end{pmatrix}}{\overset{\shortparallel}{N_2}}, \underset{\begin{pmatrix}1&1\\0&1\end{pmatrix}}{\overset{\shortparallel}{N_3}} \text{ are in } \left\{ \begin{pmatrix}a&b\\c&d\end{pmatrix} : a+b+c = 2d \right\} \right)$

<u>Def</u> A subset $S \subset V$ is "lin. dep" if it is "wasteful".
I.e., If $\exists\, a_i \in F$ not all 0 & $u_i \in S$ s.t. $\sum a_i u_i = 0$.
Otherwise, it is "lin. indep."

<u>Examples</u> $\{e_i\}$ ✓, $\cancel{\left\{ \begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}, \begin{pmatrix}7\\8\\9\end{pmatrix} \right\}}$

<u>Comments</u> 1. $\emptyset$ is lin. indep.
2. $\{u\}$ is lin indep iff $u \neq 0$.
3. Suppose $S_1 \subset S_2 \subset V$. Then
   a. If S_1 is dep, so is S_2
   b. If S_2 is indep, so is S_1
4. If S is lin indep in V and $v \in V$, Then
   $S \cup \{v\}$ is lin. dep. iff $v \in \mathrm{Span}(S)$.