13-Aarhus Day 5: Start of the w-story, to Alexander

June-02-13
4:01 AM

Optimistic Rough Tentative Plan. Fourth introduction: Dalvit on 4D knots. Then w-tangles to the Alexander polynomial.

Pasted from <http://www.math.toronto.edu/~drorbn/Talks/Aarhus-1305/>

- Dalvit on 4D knots.
- FT, TC, 4T, \alpha.
- Z.
- Bracket rise.
- wA is a bi-algebra, Milnor-Moore, primitives and group-like elements.
- wA, wB, dimensions.
- Low algebra, especially the 2D Lie Alg..
- Commutators commute, hairy Y's and detached wheels, the Alexander theorem.
- A word on BCH and the Euler trick.

on board: Regrets:

1. $W_{\mathfrak{gl}(N)}$ 2. $Z/$KTG's

3. The relationship between KTGs & associators

4. grrr.

3.6.3. *Example: The 2 Dimensional Non-Abelian Lie Algebra.* Let $\mathfrak{g}$ be the Lie algebra with two generators $x_{1,2}$ satisfying $[x_1, x_2] = x_2$, so that the only non-vanishing structure constants b_{ij}^k of $\mathfrak{g}$ are $b_{12}^2 = -b_{21}^2 = 1$. Let $\varphi^i \in \mathfrak{g}^*$ be the dual basis of x_i; by an easy calculation, we find that in $I\mathfrak{g}$ the element φ^1 is central, while $[x_1, \varphi^2] = -\varphi^2$ and $[x_2, \varphi^2] = \varphi^1$. We calculate $\mathcal{T}_\mathfrak{g}^w(D_L)$, $\mathcal{T}_\mathfrak{g}^w(D_R)$ and $\mathcal{T}_\mathfrak{g}^w(w_k)$ using the "in basis" technique of Equation (19). The outputs of these calculations lie in $\mathcal{U}(I\mathfrak{g})$; we display these results in a PBW basis in which the elements of $\mathfrak{g}^*$ precede the elements of $\mathfrak{g}$:

$$\begin{aligned}
\mathcal{T}_\mathfrak{g}^w(D_L) &= x_1\varphi^1 + x_2\varphi^2 = \varphi^1 x_1 + \varphi^2 x_2 + [x_2, \varphi^2] = \varphi^1 x_1 + \varphi^2 x_2 + \varphi_1, \\
\mathcal{T}_\mathfrak{g}^w(D_R) &= \varphi^1 x_1 + \varphi^2 x_2, \\
\mathcal{T}_\mathfrak{g}^w(w_k) &= (\varphi^1)^k.
\end{aligned}$$

m		See Section 7.5: 0	1	2	3	4	5	6	7	Comments
$\dim \mathcal{G}_m \mathcal{A}^-(\uparrow)$	$u \mid v$	$1 \mid 1$	$1 \mid 2$	$2 \mid 7$	$3 \mid 27$	$6 \mid 139$	$10 \mid ?$	$19 \mid ?$	$33 \mid ?$	$1 \mid 2$
	w	1	2	4	7	12	19	30	45	3, 4
$\dim \mathcal{G}_m \mathcal{L}ie^-(\uparrow)$	$u \mid v$	$1 \mid 1$	$1 \mid 2$	$2 \mid 7$	$3 \mid 27$	$6 \mid \geq 128$	$10 \mid ?$	$19 \mid ?$	$33 \mid ?$	$1 \mid 5$
	w	1	2	4	7	12	19	30	45	6
$\dim \mathcal{G}_m \mathcal{A}^{r-}(\uparrow)$	$u \mid v$	$1 \mid 1$	$0 \mid 0$	$1 \mid 2$	$1 \mid 7$	$3 \mid 42$	$4 \mid ?$	$9 \mid ?$	$14 \mid ?$	$1 \mid 7$
	w	1	0	1	1	2	2	4	4	3, 8
$\dim \mathcal{G}_m \mathcal{P}^-(\uparrow)$	$u \mid v$	$0 \mid 0$	$1 \mid 2$	$1 \mid 4$	$1 \mid 15$	$2 \mid 82$	$3 \mid ?$	$5 \mid ?$	$8 \mid ?$	$1 \mid 9$
	w	0	2	1	1	1	1	1	1	3
$\dim \mathcal{G}_m \mathcal{A}^-(\bigcirc)$	$u \mid v$	$1 \mid 1$	$1 \mid 1$	$2 \mid 2$	$3 \mid 5$	$6 \mid 19$	$10 \mid 77$	$19 \mid ?$	$33 \mid ?$	$1 \mid 10$
	w	1	1	1	1	1	1	1	1	3
$\dim \mathcal{G}_m \mathcal{A}^{r-}(\bigcirc)$	$u \mid v$	$1 \mid 1$	$0 \mid 0$	$1 \mid 0$	$1 \mid 1$	$3 \mid 4$	$4 \mid 17$	$9 \mid ?$	$14 \mid ?$	$1 \mid 10$
	w	1	0	0	0	0	0	0	0	3

$$Z(K) = \underbrace{\exp_{\mathcal{A}^w}\left(sl_L(K)D_L\right)\cdot\exp_{\mathcal{A}^w}\left(sl_R(K)D_R\right)}_{\text{minor part: self linking coded in arrows}}\cdot\underbrace{\exp_{\mathcal{A}^w}\left(-w\left(\log_{\mathbb{Q}[\![x]\!]} A(K)(e^x)\right)\right)}_{\text{main part: Alexander coded in wheels}},$$