

claim If $M = (m_1 | m_2 | \dots)$ is an $n \times n$ matrix and A is another, then

$$(Am_1 | m_2 | \dots) + (m_1 | Am_2 | \dots) + \dots = \det(M) \cdot \text{tr}(A)$$

"Indirect" proof: Both sides are multilinear and anti-symmetric in the columns of M , so the equality is true, up to a scalar. The scalar can be fixed by taking $M = I$.