Qualitative Analysis Scratch

November-25-12
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Changing the dependent variable.

$$y''+py'+qy \,/.\, y\to \mu V =$$

$$= \mu''V + 2\mu'V' + \mu V'' + p\mu'V + p\mu V' + q\mu V$$

$$= \mu V'' + (2\mu'+p\mu)V' + (q\mu + p\mu' + \mu'')V$$

now assume $2\mu'+p\mu=0$, get

$$= \mu V'' + \left(q\mu - \frac{p^2}{2}\mu - \left(\frac{p}{2}\mu\right)'\right)V$$

$$= \mu V'' + \left(q\mu - \frac{p^2}{2}\mu - \frac{p'}{2}\mu + \frac{p}{2}\frac{p}{2}\mu\right) = 0$$

$$\Longleftrightarrow \quad V'' + \left(q - \frac{p^2}{4}\mu - \frac{p'}{2}\mu\right) = 0$$

Changing the independent variable.

$$y''+py'+qy \quad ; \quad z = \nu(x)$$

$$\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx} = \frac{dy}{dz}\nu'$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dz}\nu'\right) = \frac{d}{dx}\left(\frac{dy}{dz}\right)\nu' + \frac{dy}{dz}\nu'' = (\nu')^2\frac{d^2y}{dz^2} + \nu''\frac{dy}{dz}$$

So eqn becomes:

$$(\nu')^2\left(\frac{d^2y}{dz^2}\right) + (\nu''+p\nu')\frac{dy}{dz} + qy = 0$$

if $\nu''+p\nu'=0$, this is is $\dfrac{d^2y}{dz^2} + \dfrac{q}{(\nu')^2}y = 0$

$x = 1/t. \qquad t = 1/x$

$$y' = \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = -\frac{1}{x^2}\dot{y} = -t^2\dot{y}$$

$$y'' = -t^2(-t^2\dot{y})^{\cdot} = t^2(t^2\dot{y})^{\cdot} =$$

$$= 2t^3\dot{y} + t^4\ddot{y}$$

So $y''+py'+qy$ becomes

$$t^4\ddot{y} + (2t^3 - pt^2)\dot{y} + qy = 0$$

this is a RSP if

$$\underset{\substack{\shortparallel \\ 2-xp}}{2-\frac{p}{t}} \quad \& \quad \underset{\substack{\shortparallel \\ qx^2}}{\frac{q}{t^2}} \text{ are analytic.}$$