Hour 33 Scratch

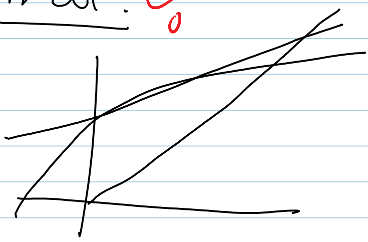
November-30-12
4:44 AM

Theorem 3.3. *If $q(x)$ is continuous and $q(x) > 0$ for all $x \geq A$ and if $\int_A^\infty xq(x)dx < \infty$, then for any solution of $y'' + qy = 0$ there is a constant K such that*

$$\lim_{x\to\infty} \frac{y(x)}{x} = K = \lim_{x\to\infty} y'(x)$$

(and in particular, no solution of $y'' + qy = 0$ is oscillatory).

Proof.



rough

y is below any one of its tangents

$\Rightarrow \exists L$ s.t. for sufficiently large x, $y(x) \leq Lx$

$$y'(b) - y'(a) = \int_a^b y'' = \int_a^b qy \leq \int_a^b L\cdot x\cdot q \to 0$$

So $y'(x)$ is a Cauchy function...

The rest is L'Hôpital...

Can I have $\lim y'(x) = 0$?

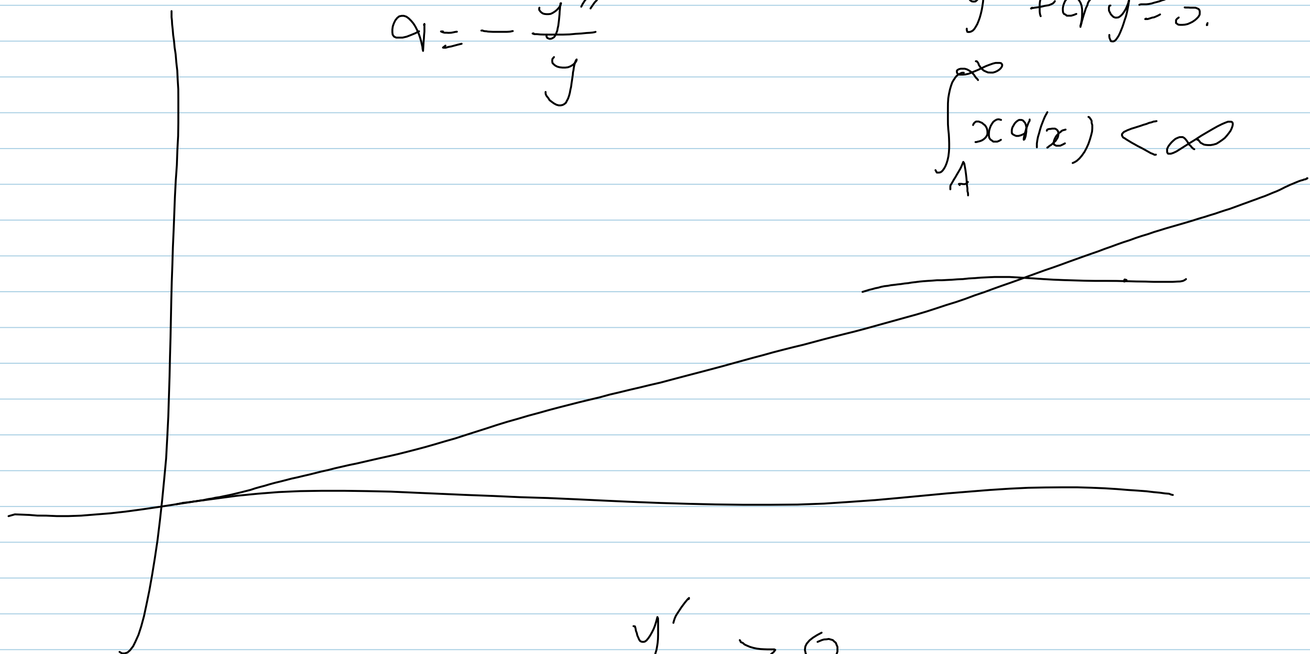
$$\lim \frac{y(x)}{x} = 0$$

$q > 0$

$y'' + qy = 0.$

$$q = -\frac{y''}{y}$$

$$\int_A^\infty xq(x) < \infty$$



$$\frac{y'}{y} > 0$$

if y' is bndd

if $\frac{y'}{y}$ is bndd away from 0, we're probably done.
FALSE for $y=x$.

$$\left(\frac{y'}{y}\right)' = \frac{y''y-(y')^2}{y^2} = \frac{-qy^2-(y')^2}{y^2}$$

$v' = -q - v^2$ so v is decreasing.

$$\frac{1/x}{\log x} = \frac{1}{x\log x}$$

$$\left(x-\frac{1}{x}\right)'' = \left(1+\frac{1}{x^2}\right)' = \frac{-2}{x^3} \qquad q = \frac{2/x^3}{x+\frac{1}{x}} = \frac{2}{x^4+x^2}$$

$$\frac{y'}{y} > \epsilon \Rightarrow y' > \epsilon y$$