Hour 27 Scratch

November-16-12
7:37 AM

$$\phi' = f(x, \phi) \qquad \phi(x_0) = y_0 \qquad \phi = \sum_{k \geq 0}^{\infty} a_k (x - x_0)^k$$

Differentiate both sides $(k-1)$ times, evaluate at x_0:

$$k!\, a_k = \frac{d^{k-1}}{dx^{k-1}} f(x, \phi(x)) \Big|_{x = x_0} =$$

$$= \frac{d^{k-1}}{dx^{k-1}} f\Big(x, \underbrace{\sum_{j=0}^{k-1} a_k (x - x_0)}_{\phi_{k-1}}\Big) \Big|_{x = x_0}$$

$$\phi_k = \phi_{k-1} + \frac{x^k}{k!} \frac{d^{k-1}}{dx^{k-1}} f(x, \phi_{k-1}) \Big|_{x = x_0}$$