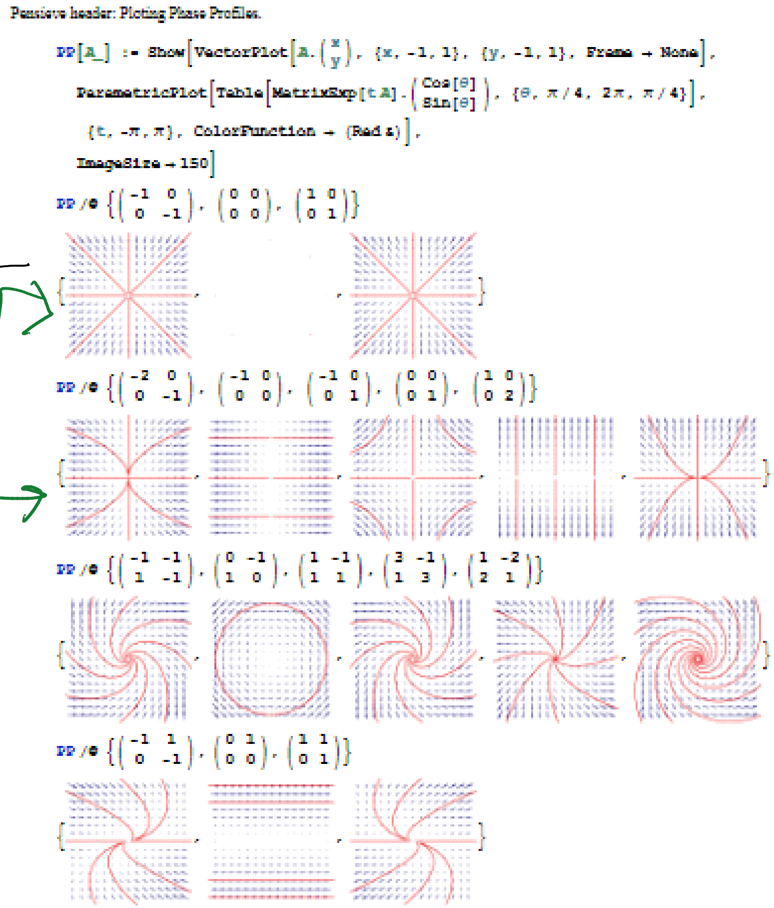
267ODEs-121102, Hour 23: Phase Portraits

October-30-12
8:02 AM

HW6 on web by midnight.

Riddle along: A game: Player A writes the numbers 1-18 on the faces of three blank dice, to her liking. Player B takes one of the 3 dice. Player B takes one of the remaining two, and throws away the third. Player A and B then play 1,000 rounds of "dice war" with the dice they hold. Whom would you rather be, player A or player B?

Phase portraits: First philosophy, then follow handout.



Cases. I Two different real eigenvalues

$\lambda_1 < \lambda_2 < 0$

$\lambda_1 \leq \lambda_2 = 0$

$\lambda_1 < 0 < \lambda_2$

$\lambda_1 = 0 < \lambda_2$

$0 < \lambda_1 < \lambda_2$

II complex eigenvalues.

$Re(\lambda) > 0$ $Re(\lambda) = 0$ $Re(\lambda) < 0$

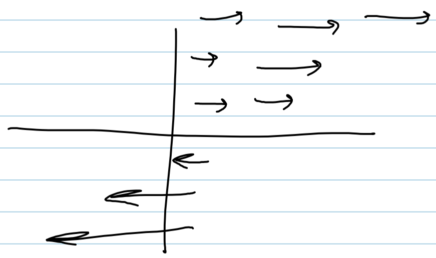
III One real eigenvalue w/ two eigenvectors

$\lambda > 0$ $\lambda = 0$ $\lambda < 0$

IV One real eigenvalue, just one eigvec — most interesting!

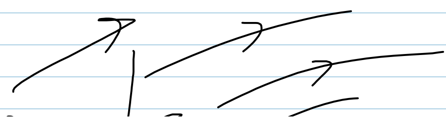
$$\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \rightsquigarrow \begin{pmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{pmatrix}$$

$\lambda = 0$:

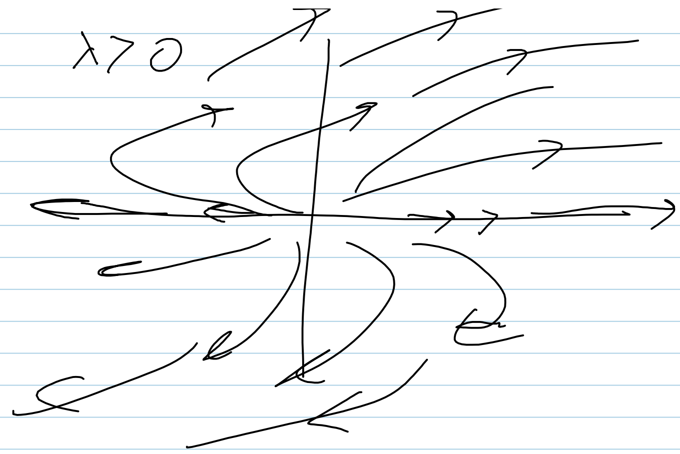


$$\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$$

$\lambda > 0$



$(e^{\lambda t}(x_0 + t y_0))$



$$\begin{pmatrix} e^{\lambda t}(x_0 + t y_0) \\ e^{\lambda t} y_0 \end{pmatrix}$$

Now the quadratic case!

done link

The non-homogeneous case, using diagonalization.

$V' = AV + g(t)$ set $V = C \cdot u$ $u = C^{-1} V$

$Cu' = AC u + g(t)$

$u' = C^{-1} A C u + C^{-1} g(t) = D u + C^{-1} g$

if $C^{-1} A C = D$ is diagonal, this is a decoupled system.

Example $V' = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix} V + \begin{pmatrix} t^{-1} \\ 2t^{-1} + 4 \end{pmatrix}$

$D = \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix}$ $C = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$ $C^{-1} = \frac{1}{5} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

$u' = \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix} u + \frac{1}{5} \begin{pmatrix} 5/t + 8 \\ 4 \end{pmatrix}$

$u_1' = 1/t + \frac{8}{5}$ $u_1 = \log t + \frac{8}{5} t + C_1$

$u_2' = -5u + 4/5$ $u_2 = \frac{4}{25} + C_2 e^{-5t}$

$V = C \cdot u = \ldots$