TT. Friday Oct. 26 9-10 @ GB404
... a partial sample test is on web!

Read Along. BDP 3.1, 3.3-3.5, 7

Riddle Along: 1 2 3 4 5 6 7 8 9

Two players alternate drawing cards from the above deck. The first player to have 3 cards that add up to 15, wins. Would you like to be the first to move or the second? (More on today's web, including a video link).

Reminder: $P \in \mathbb{R}[\alpha]$, $D = \frac{d}{dx}$, solve $P(D)\phi = 0$;

expect an n-dim v.s. of sol'ns.

$$P(D)e^{\alpha x} = P(\alpha)e^{\alpha x}$$

If p has n distinct roots, real or complex, ok.

If p has a multiple root; i.e.

$$y'' - 2y' + y = 0 \ \ldots$$

Differentiate $P(D)e^{\alpha x} = P(\alpha)e^{\alpha x}$ w.r.t. α:

$$P(D)(xe^{\alpha x}) = (P'(\alpha) + xP(\alpha))e^{\alpha x}$$

$$P(D)(x^2e^{\alpha x}) = (P'' + 2xP' + x^2P)e^{\alpha x}$$

$$\vdots$$

Alternatively, "reduction of order": If you know one solution of a 2^{nd} order homogeneous linear ODE, finding the 2^{nd} reduces to a 1^{st} order h.l. ODE:

$$y'' + py' + qy = 0. \quad p, q \text{ functions}, \quad y_1 \text{ a sol'n.}$$

Try $y = y_1 \cdot v$, get

$$\cancel{y_1'' v} + 2y_1' v' + y_1 v'' + \cancel{p y_1' v} + p y_1 v' + \cancel{q y_1 v} = 0$$

1^{st} order h.l. ODE for v'.

Exercises. 1. Check that this give the same answers.

2. How is this related to the algebra "reduction of order" for alg. eqn's?

Non homogeneous eqn's by "undetermined coeffs":

Examples. 1. $y'' - 3y' - 4y = 2\sin x$ $\qquad \alpha_{1,2} = 4, -1$

Sol'n $y = \frac{1}{17}(3\cos x - 5\sin y)$

done line

2. $y'' - 3y' - 4y = 4x^2$ Sol'n $y = -x^2 + \frac{3}{2}x - \frac{13}{8}$

3. $y'' - 4y = xe^x + xe^{2x}$

xe^x: no problem.

xe^{2x}: guess $(Ax^2 + Bx)e^{2x}$

In general, this works if RHS is a polynomial times an "exponential".

Even better, do systems: $y' = Ay \quad y(0) = y_0$

Sol'n $y(x) = e^{Ax} \cdot y_0$

What's e^{Ax}?

continue!